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Excitation of a Half-space by a Radial Current Sheet Source

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Abstract—Some advantages and problems of the new geoelectrical prospecting method, i.e., vertical electric current soundings (VECS) are discussed. This method is based on using a new source, namely a circular electric dipole (CED). The source is installed by one of the transmitter poles grounded in the central point and the other pole uniformly grounded around with a radius determined by the depth of investigation desired. It can be defined as a noninductive source. The previous research was based on the diffusion approach. In this paper the author uses the solution with due regards for displacement currents in the frequency and time domain. A major disadvantage of the CED scheme is the need to provide a symmetrical grounding of the outer ring electrode. A possible way to avoid this requirement is to adopt an ungrounded CED array.

Key words: Electrical prospecting, vertical electric current soundings, circular electric dipole.

Introduction

Sometime ago a qualitatively new controlled source, a Circular Electric Dipole (CED), was described (MOGILATOV, 1992). It may be defined as a source having no magnetic field of its own at the earth's surface. Such a geometry of conductors with a current on the surface of the earth was proposed to reduce the magnetic field of each separate conductor. In other words, CED is a noninductive source.

The source is installed by grounding one of the transmitter poles in the central point. The other pole is uniformly grounded around with a radius determined by the depth of investigation desired (Fig. 1). Let us briefly illuminate the most interesting CED properties in the low frequency regime. CED is a source having no magnetic field of its own. Thus it is a pure galvanic source, which differs from a loop (a pure inductive source) and from a line. It is both galvanic and inductive (a "line" here refers to a cable or insulated wire grounded at its end points). The normal magnetic field on the earth's surface (and above it) of a horizontally layered medium is absent (within the quasi-static approximation), and only a radial electric component exists. A CED field is at right angles to a loop field and has an azimuthal symmetry (seen ideally. The real CED array has azimuthal periodicity).

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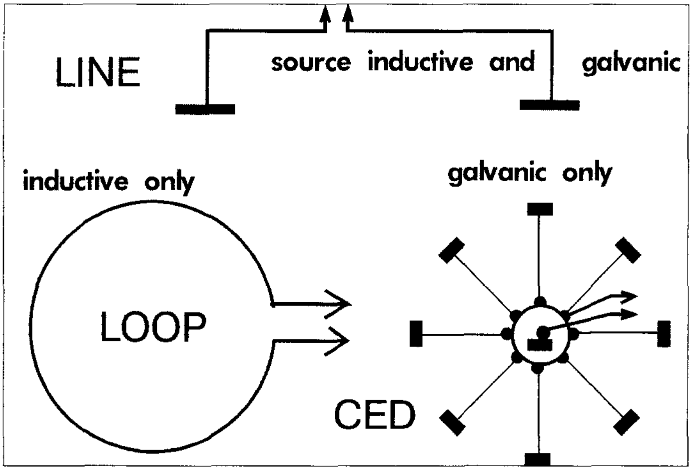

Figure 1
Three types of sources for the transient soundings.

The CED field is always defined by a vertical medium structure, at the latter transient stage as well, rather than by the total longitudinal conductivity. There is one interesting result: In marine electric prospecting a sea-water layer will not play such a fatal role when a CED is used as in applying a loop or a line. In medium with nonconducting basement the decay of the CED field is exponential. The transient process is faster than the transient process from a loop or a line. The CED can be also considered as a ground analogue of another known source namely, a vertical electric line. Finally we note that the CED as a pure galvanic source does not excite a long-term transient. Thus in all likelihood it will appear to be a new useful means to study IP processes. The CED is a *simple* source, whereas a line is *complex*. This fact should be kept in mind in studying such a complicated phenomenon as IP (Induced Polarization).

Considering a distinct vertical character of the currents under the central electrode and current circulation in vertical planes, we suggest an electric prospecting method using a CED to be named the method of vertical electric current soundings (VECS). There are the first field-test results for VECS. These assertions and previous research were based on the diffusion approach. In this paper we use the solution with due regard for displacement currents.

Field of CED in the Frequency Domain[2]

The model we adopt is very simple. As indicated in Figure 2 the radial current sheet $j_r(r)$ in A/m locates in the interface between two homogeneous half-spaces.

[2] In fact this section is a private communication from James R. Wait, reproduced with his permission.

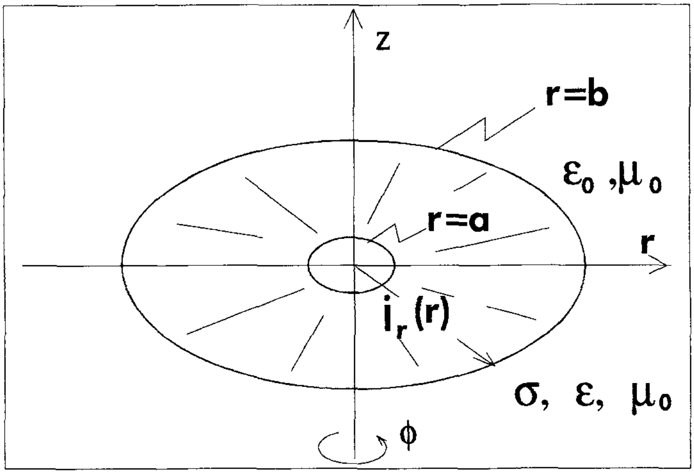

Figure 2
The model.

The upper region, for $z > 0$, which we refer to as the air, has the permittivity ε_0 and permeability μ_0. The lower lossy region, which we refer to as the earth, has the permittivity ε, conductivity σ and permeability μ_0. The objective is to deduce expressions for the fields everywhere in terms of the specified source current $j_r(r)$. A time factor $\exp(\mathrm{j}\omega t)$ is assumed where ω is the angular frequency.

Because of azimuthal symmetry, the fields can be derived from the vector potential which has only a z component. These are denoted A_0 for $z > 0$ and A for $z < 0$. Thus the nonzero field components are:

$$E_r = \frac{1}{\mathrm{j}\omega\varepsilon_0} \cdot \frac{\partial^2 A_0}{\partial r\,\partial z}, \quad z > 0; \tag{1}$$

$$E_r = \frac{1}{\sigma + \mathrm{j}\omega\varepsilon} \cdot \frac{\partial^2 A}{\partial r\,\partial z}, \quad z < 0; \tag{2}$$

$$E_z = \frac{1}{\mathrm{j}\omega\varepsilon_0} \cdot \left(-\gamma_0^2 + \frac{\partial^2}{\partial z^2}\right) A_0, \quad z > 0; \tag{3}$$

$$E_z = \frac{1}{\sigma + \mathrm{j}\omega\varepsilon} \cdot \left(-\gamma^2 + \frac{\partial^2}{\partial z^2}\right) A, \quad z < 0; \tag{4}$$

$$H_\phi = -\frac{\partial A_0}{\partial r}, \quad z > 0; \tag{5}$$

$$H_\phi = -\frac{\partial A}{\partial r}, \quad z > 0, \tag{6}$$

where $\gamma_0^2 = (\mathrm{j}\omega)^2 \varepsilon_0 \mu_0$ and $\gamma^2 = \mathrm{j}\omega\sigma\mu_0 + (\mathrm{j}\omega)^2 \varepsilon\mu_0$.

The first boundary condition is very simple. It states that E_r is continuous through the plane $z = 0$; thus

$$E_r(z = +0) - E_r(z = -0) = 0. \tag{7}$$

The second boundary condition (from Faraday's law) is that

$$H_\phi(z = +0) - H_\phi(z = -0) = -j_r(r). \tag{8}$$

To implement these conditions, we employ the following integral representations

$$A_0 = \int_0^\infty f_0(g) \cdot \exp(-u_0 z) \cdot J_0(gr) \cdot dg, \quad z > 0; \tag{9}$$

$$A = \int_0^\infty f(g) \cdot \exp(+uz) \cdot J_0(gr) \cdot dg, \quad z < 0, \tag{10}$$

where $u_0 = (g^2 + \gamma_0^2)^{1/2}$ and $u = (g^2 + \gamma^2)^{1/2}$. Notice that $J_0(gr)$ is the Bessel function of order zero, it is verified that A_0 and A satisfy Helmholtz equations $(\nabla^2 - \gamma_0^2)A_0 = 0$ and $(\nabla^2 - \gamma^2)A = 0$.

The remaining task is to determine $f_0(g)$ and $f(g)$. On using (1), (2) and (7) we deduce that

$$\frac{f_0(g)}{f(g)} = -\frac{\mathbf{j}\omega\varepsilon_0}{\sigma + \mathbf{j}\omega\varepsilon} \cdot \frac{u}{u_0}. \tag{11}$$

On the other hand, using (5), (6) and (8) we require

$$\int_0^\infty [f_0(g) - f(g)] \cdot g \cdot J_1(gr) \cdot dg = -j_r(r), \tag{12}$$

which is a Fourier-Bessel transform. Its inverse gives

$$f_0(g) - f(g) = S(g), \tag{13}$$

where

$$S(g) = \int_0^\infty j_r(r) \cdot r \cdot J_1(gr) \cdot dr. \tag{14}$$

From (11) and (13), we obtain

$$f_0(g) = \frac{-\mathbf{j}\omega\varepsilon_0 u S(g)}{(\sigma + \mathbf{j}\omega\varepsilon)u_0 + \mathbf{j}\omega\varepsilon_0 u} \tag{15}$$

and

$$f(g) = \frac{(\sigma + \mathbf{j}\omega\varepsilon)u_0 S(g)}{(\sigma + \mathbf{j}\omega\varepsilon)u_0 + \mathbf{j}\omega\varepsilon_0 u}. \tag{16}$$

On inserting these expressions into (9) and (10) we have a formally exact solution valid for any specified radial current density $j_r(r)$. Corresponding exact expressions for the field components are obtained by performing the derivative operations indicated by (1) to (6).

We now specialize the radial density to be

$$j_r(r) = I_0/(2\pi r) \quad \text{for } a \leq r \leq b, \qquad j_r(r) = 0 \quad \text{for } r < a \quad \text{and} \quad r > b, \tag{17}$$

where I_0 is the total current flowing across the annular strip. In this case

$$S(g) = \frac{I_0}{2\pi} \cdot \int_a^b J_1(gr)\, dr = \frac{I_0}{2\pi g} \cdot [J_0(ga) - J_0(gb)]. \tag{18}$$

This form for $S(g)$ would be appropriate for a pair of circular grounded electrodes of radii a and b. To preserve the assumed symmetry, they are excited by a large number of insulated wires carrying a total current I_0. Of course, if $a \to 0$ we have a point electrode at the center whence $J_0(ga) = 1$. If we further allow $gb \ll 1$, and $J_0(gb) \approx 1 - g^2b^2/4$ then

$$S(g) \approx \frac{I_0 g b^2}{8\pi}. \tag{19}$$

This appears to be a valid approximation for $b \ll r$ (i.e., the radial coordinate of the observer is substantially greater than the outer ring electrode). To simplify the subsequent discussion we assume this is the case in what follows.

For most geophysical application another simplification can be made. That is if $|\gamma_0 r| \ll 1$ (i.e., r is much less than the wavelength in air), $u_0 \simeq g$ so that the fields in the upper air region are valid solution of Laplace's equation. This is what is meant by the "quasi-static" assumption. But keep in mind, displacement currents are not ignored. However, we will say that the lower earth half-space is assumed to be well conducting in the sense that $|\sigma + \mathrm{j}\omega\varepsilon| \gg \varepsilon_0\omega$. Under these conditions (15) and (16) simplify to

$$f_0(g) \simeq -\frac{\mathrm{j}\omega\varepsilon_0 u S(g)}{(\sigma + \mathrm{j}\omega\varepsilon)g} \tag{20}$$

and

$$f(g) \simeq S(g). \tag{21}$$

Now (9) and (10) become

$$A_0 \simeq -\frac{\mathrm{j}\omega\varepsilon_0}{\sigma + \mathrm{j}\omega\varepsilon} \cdot \int_0^\infty S(g) \cdot \frac{u}{g} \cdot \exp(-gz) \cdot J_0(gr) \cdot dg \tag{22}$$

and

$$A \simeq \int_0^\infty S(g) \cdot \exp(uz) \cdot J_0(gr) \cdot dg. \tag{23}$$

If we further restrict attention to $a = 0$, and $b \ll r$ as mentioned, the above-mentioned expressions further simplify to

$$A_0 \simeq -\frac{I_0 b^2}{8\pi} \cdot \frac{\mathrm{j}\omega\varepsilon_0}{\sigma + \mathrm{j}\omega\varepsilon} \cdot \int_0^\infty u \cdot \exp(-gz) \cdot J_0(gr) \cdot dg \tag{24}$$

and

$$A \simeq \frac{I_0 b^2}{8\pi} \cdot \int_0^\infty g \cdot \exp(uz) \cdot J_0(gr) \cdot dg. \tag{25}$$

When $z = 0$ the integral in (24) can be expressed in closed form as

$$\begin{aligned}\int_0^\infty u \cdot J_0(gr) \cdot dg &= \int_0^\infty (g^2 + \gamma^2) \cdot u^{-1} \cdot J_0(gr) \cdot dg \\ &= \left[\gamma^2 - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right] \int_0^\infty u^{-1} \cdot J_0(gr) \cdot dg \\ &= \left[\gamma^2 - \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right] I_0\left(\frac{\gamma r}{2}\right) K_0\left(\frac{\gamma r}{2}\right),\end{aligned} \tag{26}$$

where I_0 and K_0 are modified Bessel functions of order 0. While the differentiations can be carried out, it will suffice to note fact the integral can be approximated by γ/r when $\gamma r \gg 1$. When $z > 0$ the corresponding approximation is

$$\int_0^\infty u \cdot \exp(-gz) \cdot J_0(gr) \cdot dg \simeq \gamma \cdot \int_0^\infty \exp(-gz) \cdot J_0(gr) \cdot dg = \frac{\gamma}{(r^2 + z^2)^{1/2}}. \tag{27}$$

Thus, in this case,

$$A_0 \simeq -\frac{\mathrm{j}\omega\varepsilon_0 I_0 b^2 \eta}{8\pi (r^2 + z^2)^{1/2}}, \tag{28}$$

where

$$\eta = \gamma/(\sigma + \mathrm{j}\omega\varepsilon) = \left(\frac{\mathrm{j}\omega\mu_0}{\sigma + \mathrm{j}\omega\varepsilon}\right)^{1/2}.$$

To deal with the case $z < 0$ (i.e., within earth), we obtain rather simply from (25) that

$$\begin{aligned}A &= \frac{I_0 b^2}{8\pi} \cdot \frac{\partial}{\partial z} \int_0^\infty \frac{g}{u} \cdot \exp(uz) \cdot J_0(gr) \cdot dg \\ &= \frac{I_0 b^2}{8\pi} \cdot \frac{\partial}{\partial z}\left[\frac{\exp(-\gamma R)}{R}\right] \\ &= -\frac{I_0 b^2}{8\pi} \cdot \frac{z}{R^3} \cdot (1 + \gamma R) \cdot \exp(-\gamma R),\end{aligned} \tag{29}$$

where $R = \sqrt{r^2 + z^2}$.

This final expression for A, under the stated assumption is the same as if the source were replaced by a vertical electric dipole (VED) of current moment $(Idz)_e$ located at $z = -h$. Using the conditions $h \ll r$, $|\gamma_0 r| \ll 1$ and $|\sigma + \mathbf{j}\omega\varepsilon| \gg \varepsilon_0\omega$ (i.e., as for CED) we have for VED (WAIT, 1982):

$$A = \frac{(Idz)_e h}{2\pi} \cdot \frac{z}{R^3} \cdot (1 + \gamma R) \cdot \exp(-\gamma R), \tag{30}$$

where $z > h$. On equating (29) and (30) we see that the equivalent electric dipole moment relates to the disc current I_0 by

$$(Idz)_e h = I_0 b^2/4 \tag{31}$$

is a remarkable result. The CED is a ground analogue of a vertical electric line in the low frequency regime.

Time Domain Solution

The formal time domain solution is the Fourier transform of the equation in the frequency domain. Experience shows that the numerical Fourier transform is extremely unstable in contrast to the quasi-static solution. Our method here is to deal with the Fourier transform in an analytical manner.

We shall study the electrical field as $e_r(t)$ in the air/earth interface and the magnetic field $h_\phi(t)$ in the upper region, for $z \geq 0$, which we refer to as the air.

We use expression (29) for the vector potential in the frequency domain. Thus, from (2) and (29) we obtain for $z \leq 0$:

$$E_r(\omega) = \frac{I_0 b^2}{8\pi(\sigma + \mathbf{j}\omega\varepsilon)} \cdot \frac{\partial^3}{\partial z^2\, \partial r} \left\{ \frac{1}{R} \cdot e^{-\gamma R} \right\}. \tag{32}$$

For the step function excitation, if the transmitter current is

$$I(t) = \begin{cases} I_0, & \text{for} \quad t < 0 \\ I_0/2, & \text{for} \quad t = 0, \\ 0, & \text{for} \quad t > 0, \end{cases} \tag{33}$$

or $I(t) = I_0 \cdot [1 - U(t)]$, where $U(t)$ is the Heaviside function. The time domain solution can be represented as follows

$$e_r(t) = \bar{e}_r - \frac{I_0 b^2}{8\pi} \cdot \frac{\partial^3}{\partial z^2\, \partial r} \left\{ \frac{1}{R} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-\gamma R} \cdot \frac{\exp(\mathbf{j}\omega t)}{(\sigma + \mathbf{j}\omega\varepsilon)\mathbf{j}\omega} \cdot d\omega \right\}, \tag{34}$$

where $\bar{e}_r$ is the solution for the direct current (expression (32) for $\omega = 0$).

Now we introduce the function F defined by

$$F(t, z, g) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} \frac{\exp(-uz)}{u} \cdot e^{\mathbf{j}\omega t} \cdot d\omega = c \cdot \exp(-\beta t) \cdot I_0(\alpha\sqrt{t^2 - T^2}) \cdot U(t - T), \tag{35}$$

where $u = \sqrt{g^2 + \gamma^2}$, $\gamma^2 = \mathrm{j}\omega\sigma\mu_0 - \omega^2\varepsilon\mu_0$ and $c = 1/\sqrt{\mu_0\varepsilon}$ is the speed of light. Also in (35) I_0 is the modified Bessel function, $\alpha = [\beta^2 - c^2g^2]^{1/2}$, $\beta = \sigma/(2\varepsilon)$ and $T = z/c$ is the arrival time. The integral in (35), after the substitution $\mathrm{j}\omega = s - \beta$, is in the form of a tabulated Laplace transform (ABRAMOWITZ and STEGUN, 1966).

With help of the function F and the convolution theorem the inverse Fourier transform we obtain

$$e_r(t) = \bar{e}_r - \frac{I_0 b^2}{8\pi} \cdot \frac{\partial^3}{\partial z^2\, \partial r} \left\{ \frac{1}{R} \cdot \int_{-\infty}^{\infty} \left[-\frac{\partial F(\tau, R, 0)}{\partial R} \right] \cdot G(t - \tau) \cdot d\tau \right\}, \tag{36}$$

where

$$G(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\exp(\mathrm{j}\omega t) \cdot d\omega}{(\sigma + \mathrm{j}\omega\varepsilon)\mathrm{j}\omega} = \frac{1}{\sigma} \cdot [1 - \exp(-2\beta t)] \cdot U(t). \tag{37}$$

The expression for the $e_r(t)$, after some transformations, is represented in the form:

$$e_r(t) = \bar{e}_r + \frac{I_0 b^2}{8\pi\sigma c} \cdot \frac{\partial^3}{\partial z^2\, \partial r} \int_{-\infty}^{\infty} I_0(\beta\sqrt{\tau^2 - T^2}) \cdot U(\tau - T) \cdot \phi(\tau) \cdot U(t - \tau) \cdot d\tau, \tag{38}$$

where

$$\phi(\tau) = -\exp(-\beta\tau) \cdot \{1 + \beta\tau + \exp[-2\beta(t - \tau)] \cdot (\beta\tau - 1)\}/\tau^2.$$

Here $T = R/c \equiv \sqrt{r^2 + z^2}/c$ and we must remember the rules for differentiation and integration of the Heaviside and Dirac functions.

The final expression for the radial electric field component on the surface of earth ($z = 0$) can be represented in the following form suitable for numerical evaluation:

$$\begin{aligned} e_r(t) = \bar{e}_r + \frac{I_0 b^2}{8\pi\sigma} \cdot \frac{T^2}{r^4} \cdot \Bigg[&\int_{-\infty}^{\infty} (I^{(2)} \cdot T - I^{(1)}) \cdot U(\tau - T) \cdot \phi(\tau) \cdot U(t - \tau) \cdot d\tau \\ &+ \phi(T) \cdot \left(1 + \frac{\beta^2 T^2}{2}\right) \cdot U(t - T) - \phi'_\tau(T) \cdot T \cdot U(t - T) \\ &+ \phi(T) \cdot T \cdot \delta(t - T) \Bigg] \end{aligned} \tag{39}$$

where

$$I^{(n)} = \frac{\partial^n}{\partial T^n} I_0(\beta \cdot \sqrt{\tau^2 - T^2}), \quad T = r/c, \quad \delta(x) \text{ is the Dirac delta-function.}$$

The results of equation (39) are shown in Figure 3. The transient process begins at $t = T$ after the transmitter current is switched off at $t = 0$, where T is the arrival time through the earth ($T = r\sqrt{\mu_0\varepsilon}$). The signal decays gradually approaching its

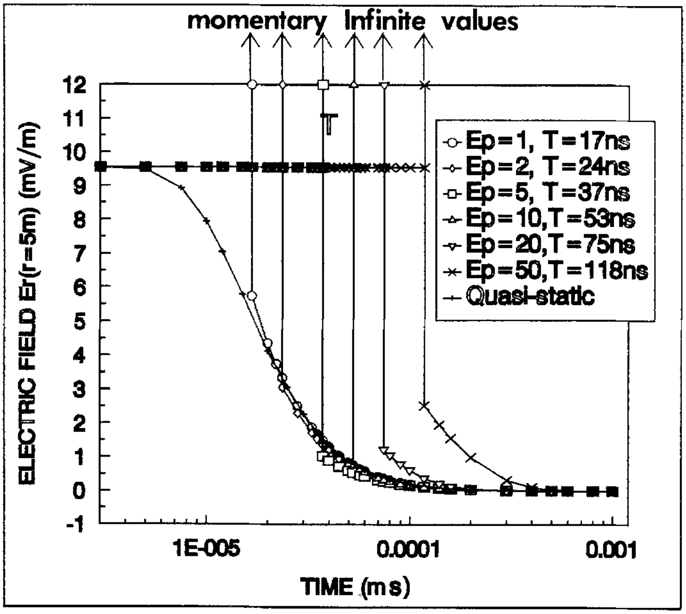

Figure 3
Transient responses in the air/earth interface (half-space, $\sigma = 0.005$ S/m) after the transmitter current is switched off. CED array, $a = 0$, $b = 0.5$ m, $I_0 = 1$ A. Distance $r = 5$ m.

diffusion behavior. At times $t < T$ we have a constant (the direct current). Precisely at time $t = T$ the field is represented by the Dirac delta-function. Notice that these momentary infinite values disappear if the current wave form is represented by any differentiable function of time or the radius of the CED has a finite value (now $b \ll r$).

The solution in the time domain for the magnetic field in the upper region, for $z \geq 0$, we obtain from (5) and (24) after Fourier transform:

$$h_\phi(t) = -\frac{I_0 b^2}{8\pi} \cdot \frac{\partial}{\partial r} \int_0^\infty e^{-gz} \cdot J_0(gr) \cdot \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\mathrm{j}\omega\varepsilon_0}{\sigma + \mathrm{j}\omega\varepsilon} \cdot \frac{g^2 + \gamma^2}{u} \cdot \frac{e^{\mathrm{j}\omega t}}{\mathrm{j}\omega} \cdot d\omega \right] \cdot dg. \quad (40)$$

We use the function F as well as the convolution theorem and the expression for the $h_\phi(t)$ can be represented in the following form

$$h_\phi(t) = -\frac{I_0 b^2 \varepsilon_0}{8\pi} \cdot \frac{\partial^2}{\partial r\, \partial t} \int_0^\infty e^{-gz} \cdot J_0(gr) \times \left[-\mu_0 F(t, 0, g) + g^2 \int_{-\infty}^{\infty} F(\tau, 0, g) G(t - \tau)\, d\tau \right] dg, \quad (41)$$

where the function G is defined in (37).

An Underground Termination

A major disadvantage of the CED scheme as described above is the need to provide a symmetrical grounding of the outer ring electrode. A possible way to avoid this requirement is to adopt an unterminated model. In this case we cannot know the distribution of the radial current density $j_r(r)$ beforehand. We propose the following way.

As indicated in Figure 4a we adopt the layered model. The source is a vertical electric line grounded in the first layer and in the lower half-space (on boundary). The first layer (is a good conductor) and a model of our array (CED) with $b = \infty$. The second layer is an insulation and the lower half-space is earth. It is the quite classical model. We obtained the expression for vector potential using Wait's book (WAIT, 1982). The bulky general expression (only a z component) is simplified if we adopt $d_a \to 0$ (the thickness of the array) and $d_i \to 0$ (the thickness of the insulation). In this case we obtain in the air ($z > 0$):

$$A_0 = -\frac{I_0}{2\pi} \cdot \frac{\mathrm{j}\omega\varepsilon_0}{S} \cdot \int_0^\infty \frac{J_0(gr) \cdot g \cdot e^{-u_0 z} \cdot dg}{\left(u_0 + \frac{\mathrm{j}\omega\varepsilon_0}{S}\right)\left(u_0^2 + \frac{u}{T(\sigma + \mathrm{j}\omega\varepsilon)}\right)}, \tag{42}$$

where $S = d_a \cdot (\sigma_a + \mathrm{j}\omega\varepsilon_a)$ is a longitudinal conductivity (in S) of CED array, $T = d_i/(\mathrm{j}\omega\varepsilon_i)$ is a transverse resistance (in $\Omega \cdot \mathrm{m}^2$) of the insulation, u_0 and u are defined as before.

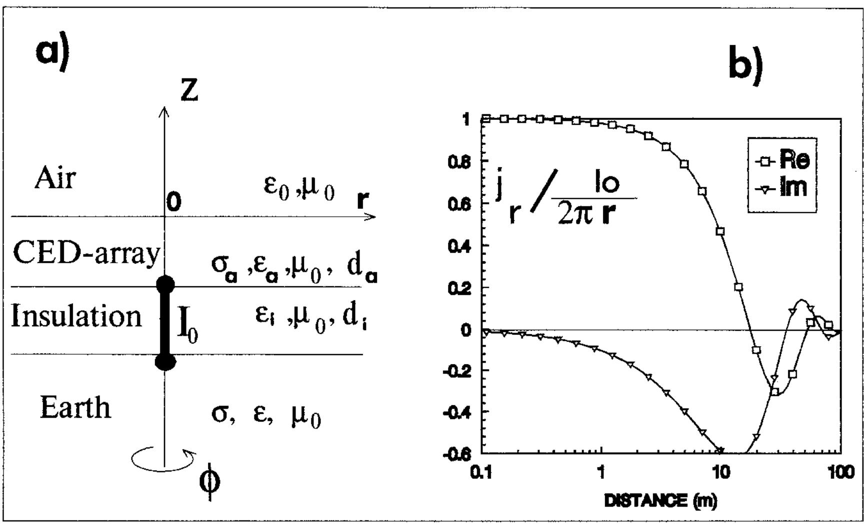

Figure 4
The model of an ungrounded CED array and the distribution of the radial current density for $f = 0.5$ MHz. $\sigma_a = 10^5$ S/m, $\varepsilon_a = \varepsilon_0$, $d_a = 0.001$ m, $\varepsilon_i = \varepsilon_0$, $d_i = 0.05$ m, $\sigma = 0.005$ S/m, $\varepsilon = 10\varepsilon_0$.

We can make another approximation, which we used before. That is if $|\gamma_0 r| \ll 1$, $u_0 \simeq g$. Besides we can adopt $S \simeq \sigma_a \cdot d_a$. Under these conditions (42) simplifies to

$$A_0 = -\frac{I_0}{2\pi} \cdot \frac{\mathbf{j}\omega\varepsilon_0}{S} \cdot \int_0^\infty \frac{J_0(gr) \cdot e^{-gz} \cdot dg}{\left(g^2 + \dfrac{u}{T(\sigma + \mathbf{j}\omega\varepsilon)}\right)}. \tag{43}$$

In this case the vector potential depends on the quality of the array (i.e., S and T). Before, for the grounded small array we neglected it. We used the boundary condition (8), where we specialize the radial current density to be $I_0/(2\pi r)$ and we rejected the secondary current density in the array.

It is very interesting to compare the distribution of the radial current density in the ungrounded array with the distribution in the grounded array. On using that E_r is continuous through the plane $z = 0$ we deduce that

$$j_r(r) = S \cdot E_r = \frac{S}{\mathbf{j}\omega\varepsilon_0} \cdot \frac{\partial^2 A_0}{\partial r\, \partial z}, \quad z = 0. \tag{44}$$

Figure 4b shows the distribution of the radial density as the ratio $j_r(r)/(I_0/2\pi r)$. The distribution can be divided into three domains. In the first domain the real part of the radial density varies with r as $1/r$ and the imaginary part is small. In the third domain $|j_r|$ is very small and it is unimportant whether the array has a grounded termination or an ungrounded termination.

The dependence of the magnetic field (namely dB_ϕ/dt) in the air on the dielectric permittivity of the earth is shown in Figure 5. It is the result of calculation of Equations (5) and (42).

Conclusions

The efficiency of a method for electric prospecting depends on several components. The choice of electromagnetic field source may be the key factor. A correct choice of source creates an optimal space-time structure for the electromagnetic field that best interacts with target objects providing real physical preconditions for solution of the problem at hand.

We propose the new method of the excitation by a radial current sheet source as an alternative to the classical methods, described as the excitation by a loop and a horizontal electrical dipole. The label "CED" (circular electric dipole) can be looked unsuccessfully sometimes. However, it is difficult to find a replacement. This label is a work name, which is conformable to labels VED, HED and VMD.

The most basic property of CED is absence of the magnetic field on the surface of the layered earth in the low frequency regime. We demonstrated that the CED is a ground analogue of another known source namely, a vertical electric dipole. These sources (CED and VED) are the noninductive, pure galvanic sources.

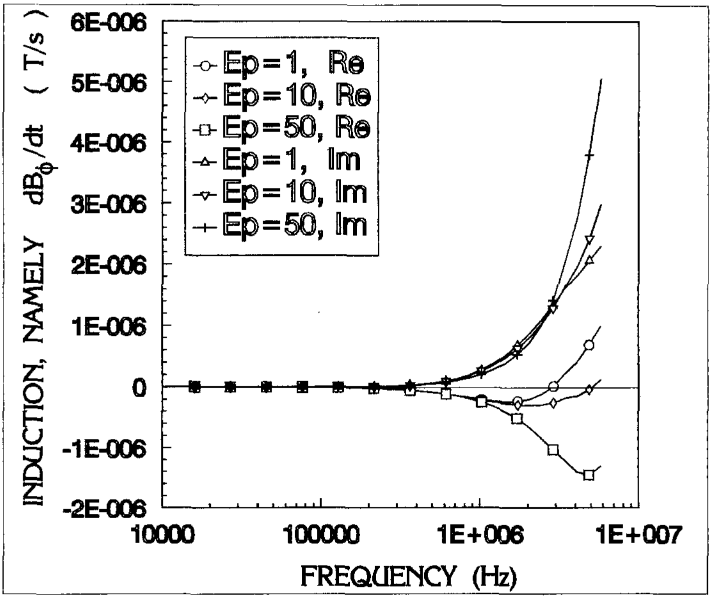

Figure 5

The magnetic induction, namely dB_ϕ/dt, in the air ($z = 0.5$ m, distance $r = 5$ m) from the CED array with the ungrounded termination for $I_0 = 1$ A. $\sigma_a = 10^5$ S/m, $\varepsilon_a = \varepsilon_0$, $d_a = 0.001$ m, $\varepsilon_i = \varepsilon_0$, $d_i = 0.05$ m, $\sigma = 0.005$ S/m, $\varepsilon = \varepsilon_0$, $10\varepsilon_0$, $50\varepsilon_0$.

A major disadvantage of the CED scheme is the need to provide a symmetrical grounding of the outer ring electrode. A possible way to avoid this requirement is to adopt an ungrounded CED array.

Up to this point our reasoning (and mathematical modeling) has been based on a view of the CED as a source with ideal azimuthal distribution of the current supplied into the medium. Actually we replace this ideal by the finite set of radially, at equal angles, arranged lines uniform in length and current. Preliminary calculations of the field from this conductor system and initial field experiments with the CED scheme have shown that requirements to the accuracy of length, angle and current can be judiciously satisfied. In the first field tests currents at rather different ground resistivities were equalized by a specially designed equipment.

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REFERENCES

ABRAMOVITZ, M., and STEGUN, I. A., *Handbook of Mathematical Functions* (Dover Publ. Inc. 1969).
MOGILATOV, V. (1992), *A Circular Electric Dipole as a New Source in Electric Survey*, J. Fizika Zemli *6*, 97–105.
WAIT, J. R., *Geoelectromagnetism* (Academic Press 1982).

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