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The Role of Displacement Currents in Transient Electromagnetic Soundings

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Abstract—The electrical prospecting by the transient electromagnetic sounding method (TEM) employs a set of the established notions. For example, it is believed that the effect of the displacement currents on the results of TEM is typically negligible except for the techniques which are intended for exploring the uppermost portions of the section and use high frequencies. In other words, for the underlying physicomathematical description, it is sufficient to use a quasi-stationary approximation. This is true for the traditional methods; however, the studies and results presented in this paper show that these notions need to be revised when TEM soundings are viewed from a more general standpoint. The suggested approach to the problem of the displacement currents' influence on TEM soundings consists in the fact that the question should be partitioned into several issues and the shallow-depth electric prospecting which employs the high frequencies (or very short transient times) should be considered separately. The problem should be viewed from the standpoint of using the TM (transverse magnetic) and TE (transverse electric) polarizations of the electromagnetic field. As the core of this paper, I suggest the new and, in my opinion, rather fascinating results showing that the displacement currents can be in principle vital for the deep electrical prospecting studies; however, this is the case only if a pure TM field is used.

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INTRODUCTION

In the electrical prospecting (primarily in a transient electromagnetic sounding method (TEM)) it is believed that the role of the displacement currents in the electromagnetic soundings of the Earth is negligible except for the shallowest techniques which use a high-frequency field. In other words, for the physicomathematical description of the problem, it is sufficient to use the quasi-stationary approximation. This is true for the traditional methods; however, our studies and results show that these notions require to be revised when TEMs are viewed from a more general standpoint. In the present work, we made an attempt to demonstrate this general view of TEM possibilities using a certain symmetric scheme shown in Fig. 1. This scheme indicates that TEM soundings in the layered Earth excited by the extraneous currents on the ground surface are possible with two polarizations of the electromagnetic field. As shown in the scheme which presents the solution of the one-dimensional (1D) problem by the method of the separation of variables, these two modes are excited by the different properties of the extraneous current distribution ($\mathrm{curl}\,\mathbf{j}_c$ and $\mathrm{div}\,\mathbf{j}_c$) (e.g., (Mogilatov, 1998)). This has long since been known in theory; however, only the TE polarization is used in practice (a pure TE mode in the case of excitation from the current loop and a predominant TE mode in the case of excitation from the AB line). At the same time, a surface source of the field that only excites the TM mode was suggested quite long ago. This is a circular electric dipole (CED), which, by the way, readily follows from the scheme itself (the radial current at $\mathrm{curl}\,\mathbf{j}_c = 0$) and has for rather a long time been used as a practical procedure in field surveys. The studies of the alternating TM field show that the common notions of TEM properties refer to the properties of the transient TE field, whereas the properties of the alternating TM field are drastically different and poorly known. We note that although the field of such a traditional source as the AB line includes the contribution of the TE mode, the field at the late stage of the transient process is dominated by the TE polarization, whereas the characteristics of the TM field remain unclear.

Thus, our approach to exploring the influence of the displacement currents on TEM suggests dividing the problem into parts and separately considering shallow electric prospecting, which uses high frequencies or very short transient times. We also suggest considering things from the standpoint of applying the TM- and TE-polarized fields. The practice is to use the TE mode (or its predominance), where the effect of the displacement currents on the electromagnetic processes is indeed negligible. This approximation has

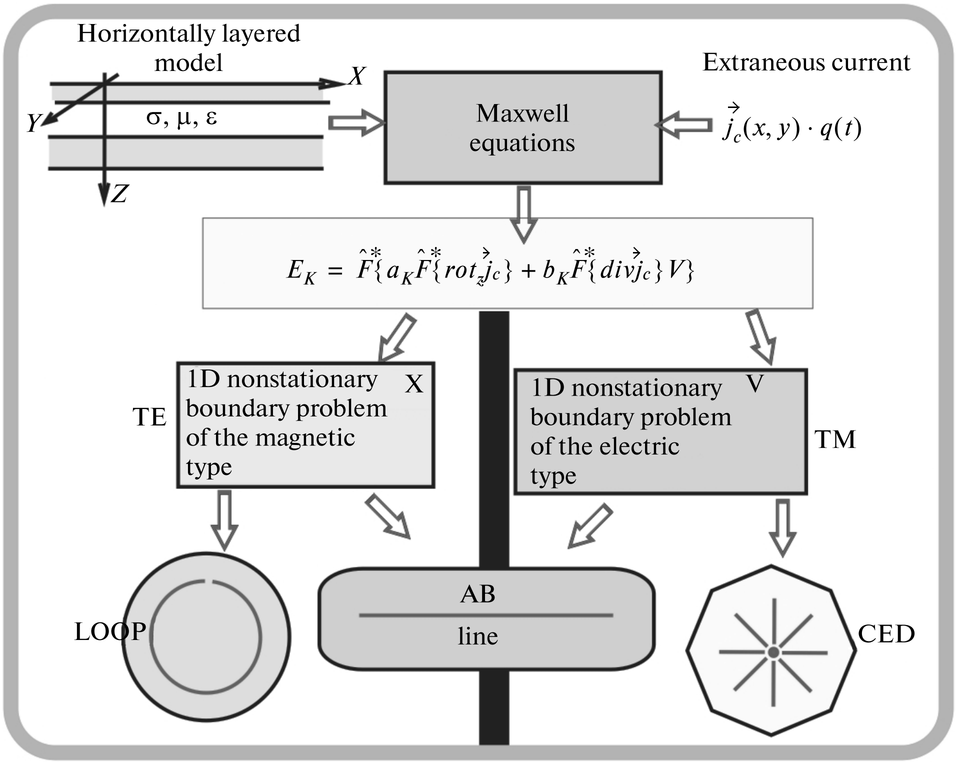

Fig. 1. The dual physicomathematical model of alternating-current electrical prospecting.

been substantiated as early as 1947 in the fundamental work (Sheinman, 1947). However, the new geoelectrical methods that employ a pure TM field have appeared since then, and the implications of the displacement currents for these methods should be studied even in the case of the deep surveys.

Below we present some theoretical considerations concerning the transient processes at super-short transient times. At this stage of the transient process, the displacement current density in Maxwell's first equation $\varepsilon \partial \mathbf{E}/\partial t$ is not small compared to the conduction current density due to the rapid variations of the field. We note that the consideration of this term changes even the type of equation for the field components. Bhattacharyya (1959) was first to suggest the full solution for the transient field problem in the model with one boundary. The subsequent attempts to make headway using analytical methods made little progress compared to advancing the quasi-stationary apparatus. As an example, we cite the solution for the media with one and two boundaries obtained in (Mogilatov, 1997) for the current loop (TE (transverse electric) mode) and in (Mogilatov, 1996) for the galvanic source of the pure TM (transverse magnetic) mode, i.e., CED. These solutions could be useful for the preliminary geophysical analysis. They can be used for testing the calculations by the more general algorithms (mesh and integral) and serve as a basis or an element of a general algorithm for the horizontally layered medium. Here we revisit these solutions for drawing a more general insight into the subject.

Finally, as the core of this paper, I present the new and, in my opinion, fascinating results which show that the displacement currents can play a vital role in the deep electric prospecting surveys, albeit, only when a pure TM-polarized field is used. We obtained the results for CED; nevertheless, these findings can also be considered in the context of the more traditional source of a TM-polarized field, i.e., a vertical electrical dipole (VED).

THE HIGH-FREQUENCY SHALLOW-DEPTH SOUNDINGS WITH A LOOP SOURCE

We consider a medium that has a single boundary ($z = 0$) which separates two homogeneous half-spaces. A vertical magnetic dipole (VMD) with moment M_z is placed in the upper half-space at the point $z = z_0$ on the vertical axis z of the cylindrical coordinates. The z-axis points vertically upwards, to the air, although at this step of the analysis we assume the arbitrary parameters of the upper and lower half-spaces. The exception is the magnetic permeability which is everywhere equal to the permeability of a vacuum. The transient field is excited by turning out the current at the zero time instant. It is required to consider Maxwell's equation, taking into account the displacement current after turning out the current (at $t > 0$). We solve this problem by separating the variables. If we follow the traditional way, we obtain the solution in the form of the Fourier convolution of the frequency-domain solution. This solution for the homogeneous half-space is well known and we do not quote it here. The consideration of the displacement currents results in the fact that in the wave number, instead of

$$k_i^2 = -\mathbf{i}\omega\mu_0/\rho_i - \omega^2\mu_0\varepsilon_i, \quad (1)$$

we have

$$k_i^2 = -\mathbf{i}\omega\mu_0/\rho_i \quad (i = 1, 2). \quad (2)$$

It may seem that this difference is insignificant. However, the allowance for the displacement currents in (1) strongly complicates the numerical implementation of the Fourier transform. A much handier and more transparent solution can be obtained by reducing the Fourier transform to the Laplace transform (by the replacement $\mathbf{i}\omega = \gamma_i - s$, $\gamma_i = 1/(2\rho_i\varepsilon_i)$, $i = 1, 2$). Let us consider the case which is topical for practical use, namely, when the VMD source is located on the surface of the homogeneous Earth. The field in the form of the components E_φ and dB_z/dt is also observed on the ground. This problem was analyzed in (Bhattacharyya, 1959); however, we obtained a different result, namely, the presence of fronts with infinite amplitudes (for the point source with a step excitation).

Thus,

$$E_\phi = \frac{M_z\mu_0}{2\pi}(A_1 - A_0), \quad (3)$$

and for A_i we have

$$\begin{aligned} A_i = \frac{\bar{\rho}}{\mu_0}\frac{T_i^2}{r^4}\Bigg[& \int_{-\infty}^{\infty}\left(I_i^{(2)}T_i - I_i^{(1)}\right)U(\tau - T)\varphi_i(\tau) \\ & \times U(t-\tau)d\tau + \varphi_i(T_i)\left(1 + \frac{\gamma_i^2T_i^2}{2}\right)U(t - T_i) \\ & - \varphi'_{i\tau}(T_i)T_iU(t - T_i) + \varphi_i(T_i)T_i\delta(t - T_i)\Bigg], \end{aligned} \quad (4)$$

where $U(x)$ is the Heaviside step function, $\delta(x)$ is the Dirac delta function, $I^{(n)}$ is the nth derivative with respect to T of the function $I_i \equiv I_i^{(0)} = I_0(\gamma_i\sqrt{\tau^2 - T_i^2}$, I_0 is the modified Bessel function, $T_i = r/c_i$ is the wave arrival time, $c = 1/\sqrt{\mu_0\varepsilon_i}$ is the speed of light in a given medium,

$$\varphi_i(\tau) = -\frac{\exp(-\gamma_i\tau)}{\tau^2} \times \{1 + \gamma_i\tau + \exp[-2\bar{\gamma}(t-\tau)](2\bar{\gamma}\tau - \gamma_i\tau - 1)\},$$

$\bar{\gamma} = 1/(2\bar{\rho}\bar{\varepsilon})$, $\bar{\varepsilon} = \varepsilon_1 - \varepsilon_0$, $\bar{\rho} = \rho_0\rho_1/(\rho_0 - \rho_1)$,,$i = 0,1$.

These expressions are suitable for the calculations. These are the general formulas for the arbitrary values of resistivity and dielectric permittivities of the upper and lower half-spaces. If the upper half-space is air, then in A_0 we should set $\gamma_0 = 0$, then

$$\begin{aligned} A_0 = \frac{\bar{\rho}}{\mu_0}\frac{T_0^2}{r^4}\Big[& \varphi_0(T_0)U(t - T_0) \\ & - \varphi'_{0\tau}(T_0)T_0U(t - T_0) + \varphi_0(T_0)T_0\delta(t - T_0)\Big]. \end{aligned} \quad (5)$$

The formula for another component which is used in practice, $\dot{B}_z$, is

$$\dot{B}_z = -\left(\frac{1}{r} + \frac{\partial}{\partial r}\right)E_\varphi. \quad (6)$$

Figure 2 presents the results of the calculations by formulas (3)–(5). We recall that the field is excited by the stepwise turning off of the current at the point source. This is very important here. The curves $E_\varphi(t)$ are calculated for the observation point at a distance of 5 m from the VMD source on the surface of the Earth whose resistivity is 100 Ω m. The codes of the curves denote the relative dielectric permeability of the Earth. Thus, at the initial time instant, the current in the dipole changes to 0. All that occurs with the component E_φ at the observation point during the time interval from 0 to ∞ is divided into three segments by two singularity points. Within the first segment, $0 < t < T_0$, the fact of turning off the source does not show itself anyhow as yet, according to the fundamental physical principle which has now been taken into account in Maxwell's equations. Correspondingly, $E_\varphi = 0$ here. At $t = T_0 = 16.678$ ns, the wave arrives at the observation point by air. At this moment, the value of E_φ builds up from 0 to ∞ and then returns to a finite level, which depends on the Earth's dielectric permittivity and resistivity. Starting from T_0, the second stage which is characterized by the smooth growth of the field commences. This process depends on the parameters of the Earth (ρ_1 and ε_1); however, it was excited by the wave that arrived by air. At time $T_1 = r\sqrt{\mu_0\varepsilon_1}$, the observation point is reached by the wave that propagates across the ground, and the value of E_φ undergoes an instantaneous change to −∞ and back to a certain finite value. Here the third stage begins, during which the field gradually diminishes and approaches a quasi-stationary solution. This stage of the process is shown in Fig. 3 in terms of the apparent resistivity curves.

The emergence of the instantaneous infinite values of the observed field is associated with the fact that we consider an abstract idealized object, i.e., a point source with an instantaneously varying moment. By the way, in the case of a real source (a loop) with a finite size, the particular location of the point where the current is supplied to the ground is also important on these time scales because the loop is not an axisymmetric source.

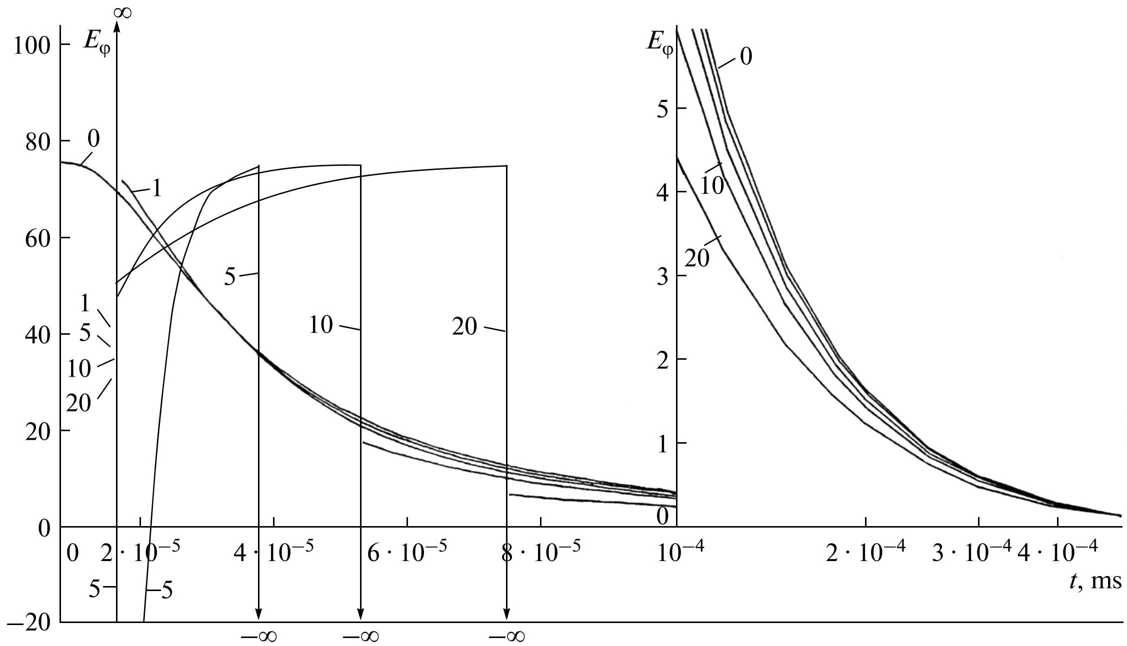

Fig. 2. The transient curves for the intensity of the electric field from a stepwise change (down to 0) of the VED moment for different values of the dielectric permeability of the Earth. The codes of the curves indicate $\varepsilon_1/\varepsilon_0$. The zero code corresponds to the quasi-stationary curve.

In (Mogilatov, 1997), the solution for two boundaries is also presented. This solution is much more complicated. Generally, allowance for the displacement currents strongly limits the possibilities of the analytical method (the method of the separation of variables). The representation of the solution in the form of the Fourier transform of the full frequency-domain solution cannot be implemented numerically without additional complicated transformations as it is carried out, e.g., in (Wait, 1982) for a two-layer medium. In practice, the experience of creating the operative (1D) software for the quasi-stationary TEM sounding surveys, which covers several decades of efforts, helps little. The primary cause here lies in the change of the type of the equation. However, the practical demands in this field were also not pressing.

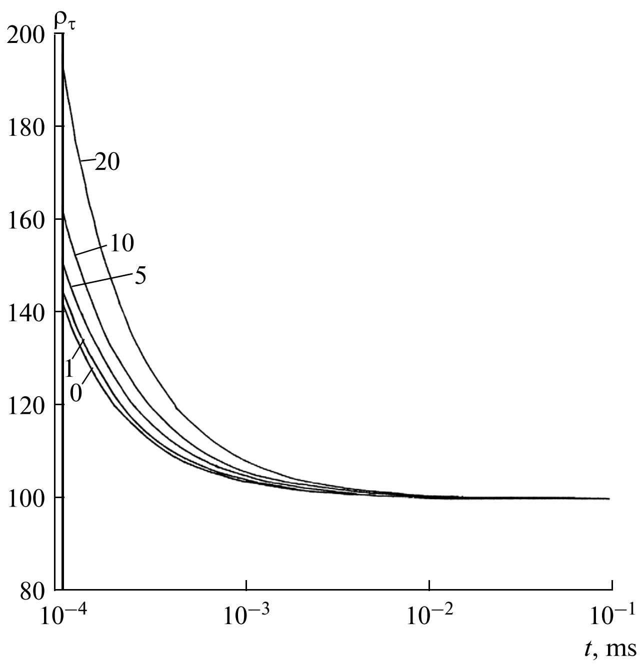

Fig. 3. The apparent resistivity curves for the different values of the dielectric permeability of the Earth. The numerical codes of the curves indicate the ratio $\varepsilon_1/\varepsilon_0$. The zero code corresponds to the quasi-stationary curve.

CED. SUPER-EARLY STAGE OF THE TM-POLARIZED FIELD

We consider the current loop that is excited in the TE-polarized field. Let us now focus on the super-early stage of the transient process for the TM-polarized field. The role of the source of the pure TM-mode can be played by a toroidal coil (providing inductive excitation), VED (providing the inductive and galvanic excitation), or a CED. The latter is a surface analog of VED (e.g., (Mogilatov, 1996)) and is used in the present analysis. The notion of CED describes a distribution of the extraneous radial current whose density is nonzero on the circle with radius r_0:

$$j_r^{\text{cm}}(r) = \frac{I}{2\pi r}[U(r - r_0 + dr_0/2) - U(r - r_0 - dr_0/2)], \quad (7)$$

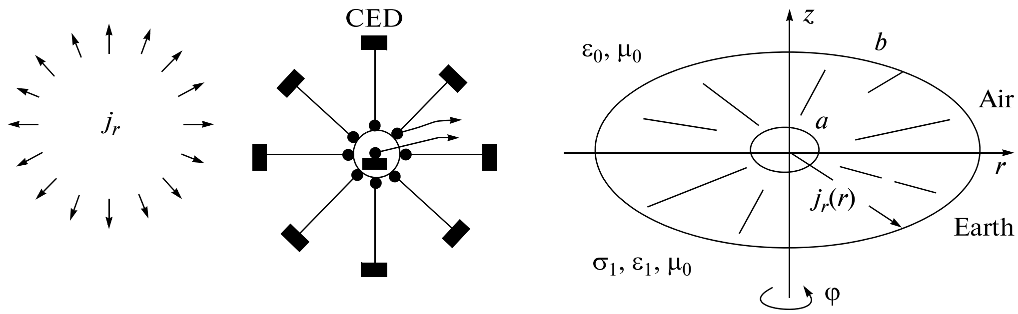

Fig. 4. The theoretical, real, and perfect CED.

where $U(x)$ is the Heaviside function (Fig. 3, left), and the azimuthally uniform radial current which is earthed along the circles with radii $a < b$ (Fig. 3, right). The case of the central earthing ($a = 0$) certainly has the highest practical value. Besides, there is a practical implementation of CED in the form of a finite set of lines (Fig. 4, central part), which has long been used in field surveys. In a 1D (horizontally layered) medium, in the cylindrical coordinates, this source excites the electrical components E_r, E_z and the magnetic component H_φ, which determines CED as the source of the TM-mode.

Let us apply for this source the same method of solution as we used for the current loop. The formula for the gradient in the frequency domain (Mogilatov, 1996) which was derived with some assumptions is (at $z \leq 0$)

$$E_r(\omega) = \frac{I_0 b^2}{8\pi\hat{\sigma}_1} \frac{\partial^3}{\partial z^2 \partial r} \left\{ \frac{1}{R} e^{-k_1 R} \right\}, \quad (8)$$

where $\tilde{\sigma}_1 = \sigma_1 + \mathbf{i}\omega\varepsilon_1$. We convert this formula to the time domain using the Fourier transform. Then, using the substitution $\mathbf{i}\omega = \gamma_i - s$, $\gamma_i = 1/(2\rho_i\varepsilon_i)$, $i = 1, 2$, we convert the Fourier transform into the Laplace transform. With the integral Laplace transform, we obtain the formula for the radial electric component of the field on the Earth's surface ($z = 0$) which is suitable for the numerical analysis:

$$\begin{aligned} E_r(t) = \bar{E}_r + \frac{I_0 b^2}{8\pi\sigma_1} \frac{T_1^2}{r^4} \Bigg[& \int_{-\infty}^{\infty} \left(I^{(2)} T_1 - I^{(1)} \right) U(\tau - T_1) \\ & \times \varphi(\tau) U(t - \tau) d\tau + \varphi(T_1) \left(1 + \frac{\gamma_1^2 T_1^2}{2} \right) U(t - T_1) \\ & - \varphi'_\tau(T_1) T_1 U(t - T_1) + \varphi(T_1) T_1 \delta(t - T_1) \Bigg], \end{aligned} \quad (9)$$

where the designations are the same as previously.

The results of the calculations by formula (9) are illustrated in Fig. 5. The transient process starts at the time instant $t = T_1$ after turning off the source current at $t = 0$, where T_1 is the arrival time of the wave that propagates across the ground ($T_1 = r\sqrt{\mu_0\varepsilon_1}$). The response decays and gradually approaches a quasi-stationary solution. At the time $t < T_1$ we have a constant (a direct current). At the time instant $t = T_1$, the strength of the electric field is infinite. We note that these instantaneous infinite values disappear when we take into account the more realistic conditions: a smooth (instead of a stepwise) change of the current in the source or a finite-size source (in the case discussed, we assumed $b \ll r$, i.e., in fact, the source has no size (is vanishingly small)).

Comparing Figs. 2 and 5, we see that the key distinction from the early stage of the transient field of the loop (the TE-polarized field) lies in the fact that the wave arriving by air is absent.

DEEP ELECTRICAL PROSPECTING

Finally, let us consider the situation of the big or deep (structural) electrical prospecting. The problem of the allowance for displacement currents in the conventional electrical prospecting, which is based on the application of a current loop and a line, has long been solved ultimately and irrevocably. The influence of the displacement currents is always negligible in the presence of the TE-polarization, which always exists in the field created by these sources. However, when considering the pure TM-polarized field, we should revisit this subject. Specifically, the question concerns the influence of a thin highly resistive horizon. In the direct-current resistivity prospecting (a constant TM-polarized field), the question is quite clear: the underlying strata are inaccessible for the study. However, in the case of the induction mode (harmonic or transient), the question is more complicated. We cannot apply the quasi-stationary approximation inside the insulator layer, and we have to take into account the displacement currents.

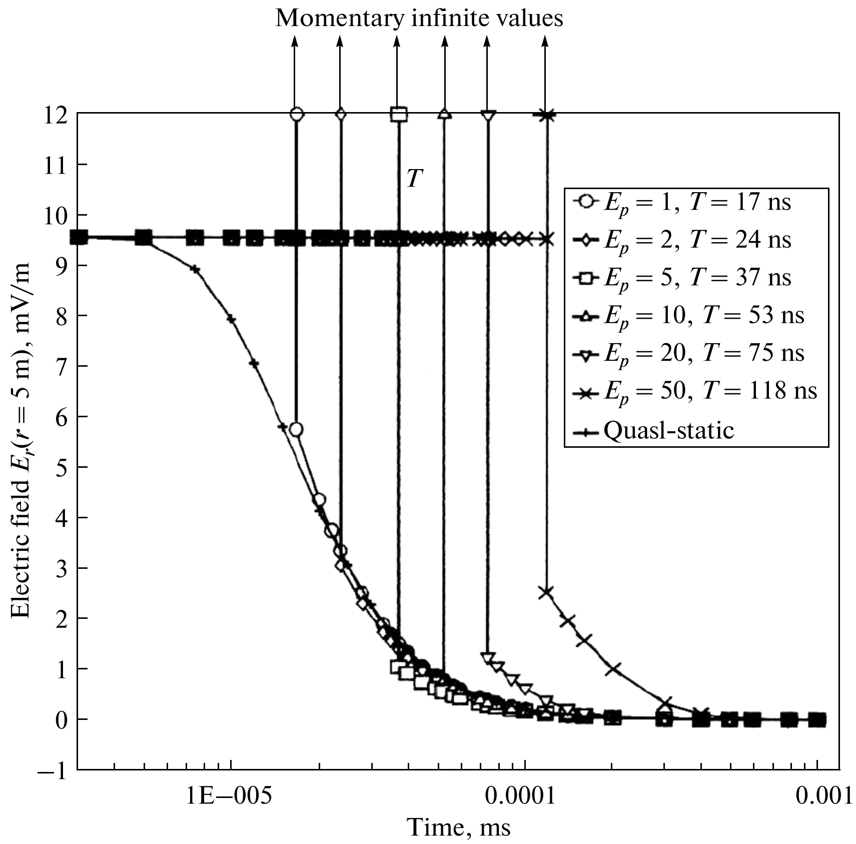

Fig. 5. The transient electromagnetic response of the medium to the excitation by CED.

We carried out a series of computations with allowance for the displacement currents for a model with an insulating thin layer shown in Fig. 6a (the CED radius was 500 m and the spacing was 2000 m) and, as it repeatedly occurred in the analysis of the behavior of the TM-polarized field, the obtained result was startling. Figure 6b compares the TEM curves for different values of dielectric permittivity of the insulating horizon. The first curve is quasi-stationary ($\varepsilon_2 = 0$) and completely determined by the upper layer. The decay of this curve is exponential, just as in any medium with the insulating basement in the case of the TM-polarized field (from VED or CED). The other curves demonstrate the radial effects of the displacement currents, including the change of the sign of the signal (E_r) followed by the power-law (t^{-2}) attenuation despite the insulating basement. Moreover, a strong dependence of the lower layer on resistivity takes place, as demonstrated in Fig. 7. Hence, the displacement currents in the discussed case have drastically interrupted the transient process.

This result was obtained by the conventional approach which consists in the Fourier transformation of the frequency-domain solution of the problem for the 1D (layered) medium. This approach was widely used for the quasi-stationary approximation; however, of course, it could lead to errors in the allowance for the displacement currents. Naturally, this result required additional validation by independent methods. We achieved this validation by solving the problem in the time domain by the method of A.N. Tikhonov.

THE SOLUTION ALLOWING FOR THE DISPLACEMENT CURRENTS IN THE TIME DOMAIN

We analyze the model of the medium which is shown in Fig. 6a. For simplicity, we set $\rho_1 = \rho_3 = \rho$, $h_1 = h_3 = h$, and assume that $h_2 \ll h_1, h_3$. Let us find the solution which takes into account the displacement currents at the late stage of the transient process, in contrast to the solution for the super-early transient time presented above. We use Tikhonov's method which was previously applied for the quasi-stationary approximation only. A necessary condition for solving the problem by Tikhonov's method lies in constraining the problem along the vertical direction, which is possible when there are the upper and lower insulating half-spaces in the medium. We retain this condition and solve the quasi-stationary problem everywhere except for the thin insulating layer (Fig. 6a).

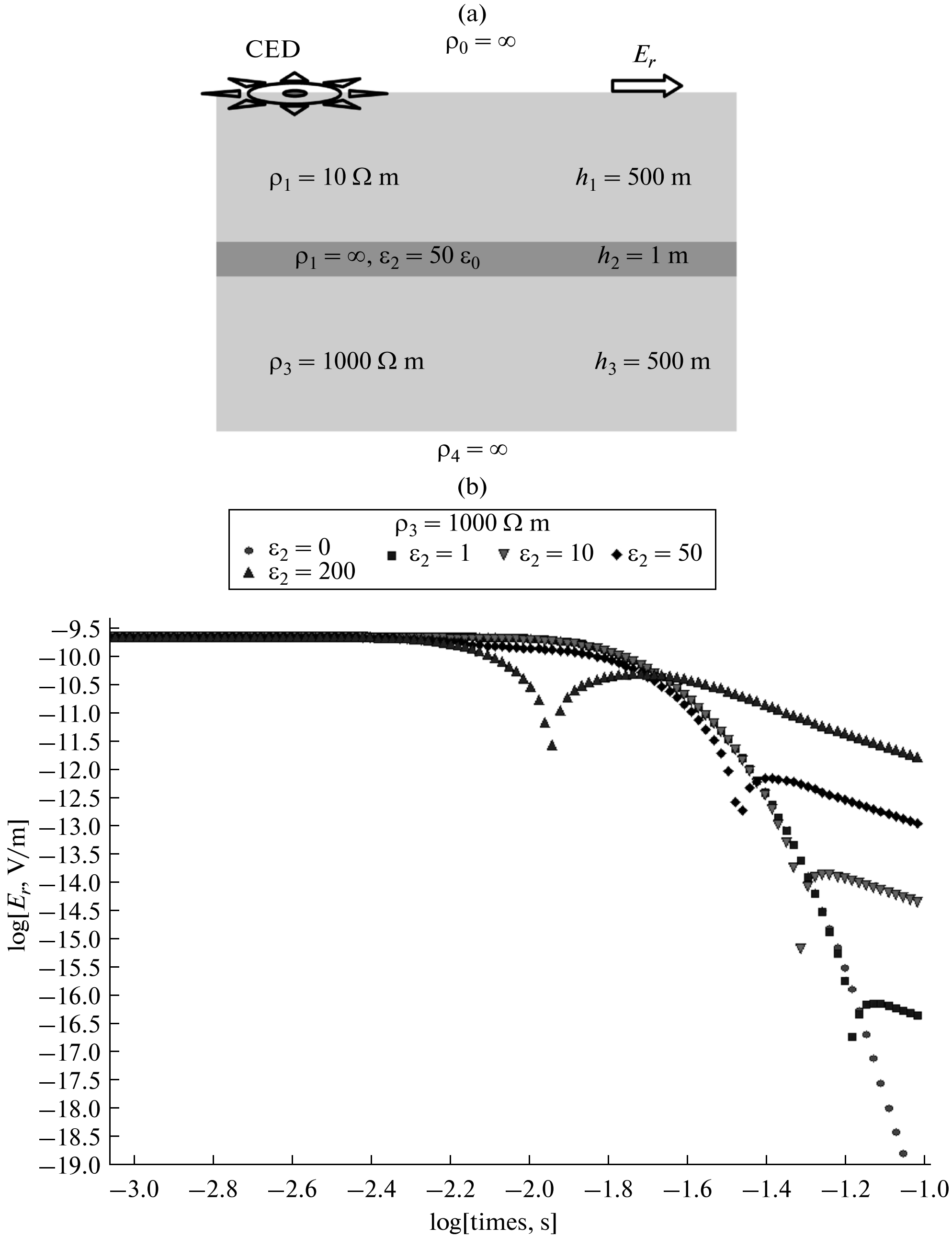

Fig. 6. The model and the transient curves for the different values of the dielectric permeability of the second layer.

We introduce the vector potential in the common way for electric source:

$$\mathbf{H} = \operatorname{curl} \mathbf{A}. \tag{10}$$

Due to the symmetry of the problem, it is sufficient to require that only the component A_z is nonzero. Then, in the cylindrical coordinate system we have

$$E_r + \varepsilon\rho\frac{\partial E_r}{\partial t} = \rho\frac{\partial^2 A_z}{\partial r \partial z}, \quad H_\varphi = -\frac{\partial A_z}{\partial r},$$
$$E_z = -\mu_0\frac{\partial A_z}{\partial t} - \mu_0\varepsilon\frac{\partial^2 A_z}{\partial t^2} + \rho\frac{\partial^2 A_z}{\partial z^2}. \tag{11}$$

Component A_z (11) should satisfy the sufficient conditions:

$$\Delta A_z = \frac{\mu_0}{\rho}\frac{\partial A_z}{\partial t} + \mu_0\varepsilon\frac{\partial^2 A_z}{\partial t^2}, \quad 0 < z < 2h,$$
$$(z \neq h), \quad 0 \le r \le \infty. \tag{12}$$

The potentials A_z and $\rho A_z'$ are everywhere continuous, which means that $A_z = 0$ at the top and bottom boundaries (thus, the problem is limited along the z-axis). Then, $A_z \to 0$ at $r \to \infty$ and $A_z \to 0$ at $t \to \infty$. The

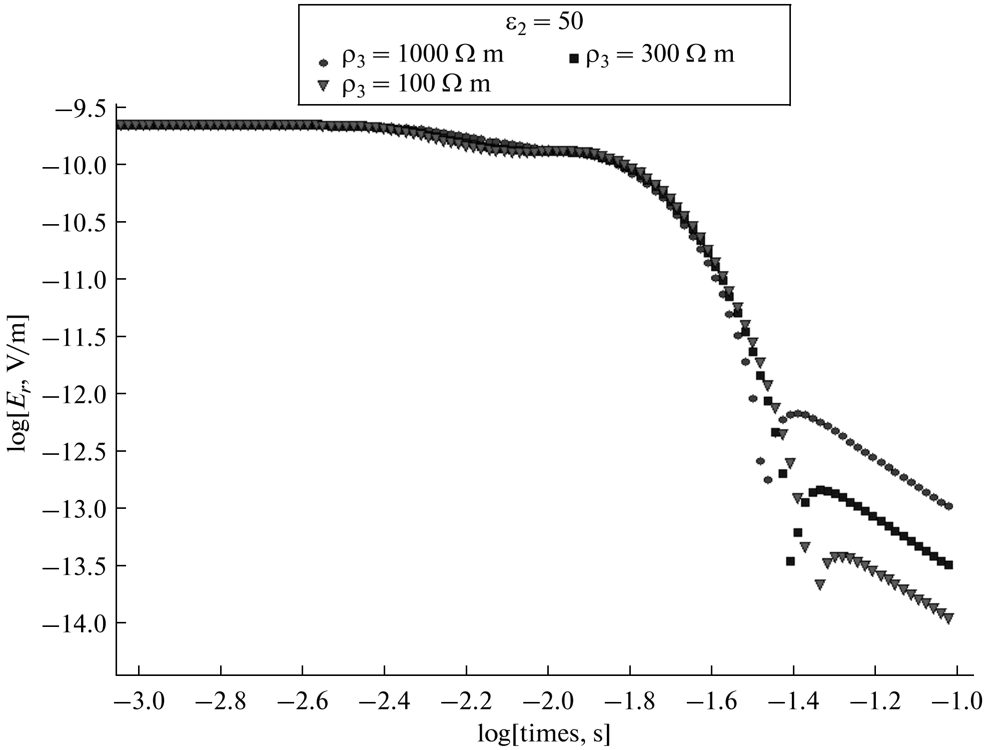

Fig. 7. The different resistivities beneath the screen.

initial condition is specified by the solution of the stationary problem.

Separating the variable r in the equation for A_z and taking into account the constraints for the potential, we search for the solution in the following form:

$$A_z = \frac{Idr_0}{4\pi}\int_0^\infty J_0(\lambda r)Z(\lambda, z)d\lambda. \quad (13)$$

The solution of the problem for Z is sought in the form

$$Z(z,t) = \sum_{j=1}^{\infty} C_j\zeta_j(z)\exp(-\alpha_j t). \quad (14)$$

For establishing the conditions of passing through the thin insulating layer, we consider the model shown in Fig. 8. The upper and lower half-spaces are conductive. The layer is insulating. The displacement currents are only taken into account within this layer. We assume that this layer is thin so that $\delta h \ll r$, $|z|$ (the coordinates of the observation point).

Within the layer, the solution expressed through the values at the upper boundary has the following form (index j is omitted):

$$\zeta(z) = \zeta_1\cosh[u(z - z_1)] + \frac{\zeta_1'}{u}\sinh[u(z - z_1)],$$
$$\zeta_z'(z) = \zeta_1 u\sinh[u(z - z_1)] + \zeta_1'\cosh[u(z - z_1)], \quad (15)$$

where $\zeta_i = \zeta(z_i)$, $\zeta_i' = \zeta_z'(z_i)$, and $u = \sqrt{\lambda^2 + \alpha^2\mu_0\varepsilon}$ (beyond the layer $u = \sqrt{\lambda^2 - \alpha\mu_0/\rho}$). Or, taking into account that $\delta h \ll r$, $|z|$,

$$\zeta_2 \cong \zeta_1 + \zeta_1'\delta h, \quad \zeta_2' \cong \zeta_1 u^2\delta h + \zeta_1'.$$

These formulas describe the situation inside the layer. Considering the conditions at the simple boundary

$$\left[\zeta'\frac{\rho}{1-\alpha\rho\varepsilon}\right] = 0, \quad [\zeta] = 0,$$

we assume that in the boundary problem, the insulating thin layer is taken into account as a boundary ($z = h$) with the transition conditions

$$[\zeta] = \zeta_1'\delta h, \quad [\zeta'] = -\zeta_1\frac{\delta h}{\alpha\rho\varepsilon}\left(\lambda^2 + \alpha^2\mu_0\varepsilon\right). \quad (16)$$

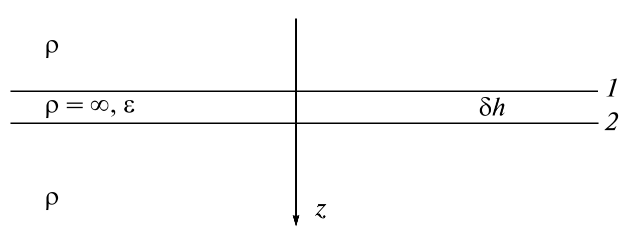

Fig. 8. The thin insulating layer.

For each term of series (14) (index j is omitted), we determine the conditions for ζ:

$$\zeta''_{zz} + u^2\zeta = 0, \quad 0 < z < 2h, \quad (z \neq h),$$
$$\zeta(0) = \zeta(2h) = 0, \quad [\zeta']|_{z=h} = -\frac{\delta h}{\varepsilon\alpha\rho} \tag{17}$$
$$\times\left(\lambda^2 + \alpha^2\mu_0\varepsilon\right)\zeta(h), \quad [\zeta]|_{z=h} = \zeta_1'\delta h.$$

In each of the domains separated by the dielectric boundaries, we obtain the solution (by passing to the trigonometric solution):

$$\zeta_1(z) = a\sin(uz), \quad 0 \le z \le h,$$
$$\zeta_2(z) = b\sin[u(z - 2h)], \quad h \le z \le 2h. \tag{18}$$

We have already taken into account the conditions at the top and bottom boundaries. Using the conditions at $z = h$, we obtain the values a, b and the equation for α:

$$s^2\frac{\delta h}{\varepsilon\alpha\rho}\left(\lambda^2 + \alpha^2\mu_0\varepsilon\right) - c^2u^2\delta h = 2csu,$$

where $u^2 = \frac{\alpha\mu_0}{\rho} - \lambda^2$, $s = \sin(uh)$, and $c = \cos(uh)$. At the long times ($t \to \infty$), integral (13) is determined at $\lambda \to 0$, the series (14) is determined by the first term, and we are interested in α_1 that tends to zero at $\lambda \to 0$ (Tikhonov and Skugarevskaya, 1950). Only $\alpha \sim \lambda^2$ is suitable, so that

$$a = 1, \quad b = -1, \quad \alpha \cong \frac{\delta h h}{2\rho\varepsilon}\lambda^2. \tag{19}$$

The coefficients in (14) can be found from the orthogonality of functions ζ_j (in the formula below, index j is omitted):

$$C = \frac{\lambda^2}{u^2}\int_0^{2h}\bar{Z}\zeta dz \Bigg/ \int_0^{2h}\zeta^2 dz. \tag{20}$$

The stationary solution which is the initial value for series (14) satisfies the boundary problem

$$\bar{Z}''_{zz} - \lambda^2\bar{Z} = 0, \quad 0 < z < h,$$
$$\bar{Z}(0) = \bar{Z}(h) = 0, \tag{21}$$
$$[\bar{Z}'_z]\Big|_{z=z_0} = 0, \quad [\bar{Z}]\Big|_{z=z_0} = 2J_1(\lambda r_0),$$

where r_0 is the radius of the source. We limit the area of the definition by the value $z = h$. Generally speaking, we should also take into account the presence of the electric field in the thin insulating layer; however, this correction is small. The solution below the source which is placed on the top boundary (but on the underside of it) is (at $r_0 \ll h, z$)

$$\bar{Z}(z) = r_0^2\lambda\frac{\sinh[\lambda(h - z)]}{2\sinh(\lambda h)}.$$

Then we calculate

$$\int_0^{2h}\bar{Z}\zeta dz = \frac{r_0^2\lambda}{\lambda^2 + u^2}\frac{\lambda\sin(uh)}{2\sinh(\lambda h)},$$

$$\int_0^{2h}\zeta^2 dz = -h\left[1 + \left(\frac{\sin(uh)}{uh}\right)^2\right].$$

At $\lambda \to 0$ (late decay times), we can obtain the following formula for the radial gradient on the ground surface:

$$E_r = \frac{Ir_0^2\rho}{16\pi h}\int_0^\infty J_1(\lambda r)\lambda^2\exp\left(-t\frac{\delta h}{2\rho\varepsilon}\lambda^2\right)d\lambda. \tag{22}$$

The following integral is known:

$$\int_0^\infty J_1(\lambda r)\lambda^2\exp\left(-\alpha\lambda^2\right)d\lambda = \frac{r}{4\alpha^2}\exp\left(-\frac{r^2}{4\alpha}\right).$$

Hence, for large t we have

$$E_r = \frac{Ir_0^2 r}{16\pi}\frac{\rho^3}{h^3}\frac{\varepsilon^2}{\delta h^2}\frac{1}{t^2}. \tag{23}$$

THE RESULTS OF TESTING

The simple formula (23) perfectly describes the late stage of the processes similar to those displayed in Figs. 6 and 7 (with the equal resistivities of the upper and lower layers). At our request, our colleagues—the experts in numerical modeling by the finite element method (FEM)—kindly conducted the corresponding calculations, which confirmed the results presented in Figs. 6 and 7 in the entire range of transient times. Here, the FEM calculations for the discussed problem required a laterally much larger computation area (a tub) than was specified for purely quasi-stationary problems. Moreover, with decreasing resistivity it was necessary to increase the size of the tub, which contradicted the quasi-stationary experience. It can be hypothesized that a wave propagates in a thin insulating layer, and the more conductive layers underlying and overlying the insulator layer promote better propagation (the waveguide effect).

Let us present a comparison of the three methods: in the frequency domain (the Fourier integral) and in the time domain (Tikhonov's method and Fourier series) for the resistivity of the upper and lower layers of 10 Ω m. The relative dielectric permittivity of the insulator layer is 50 (we use the model shown in Fig. 5a). Figure 9 shows the transient curves (E_r on the ground surface). We note that the FEM calculations in the required range of the transient times would require a large tub with a radius of 560000 m.

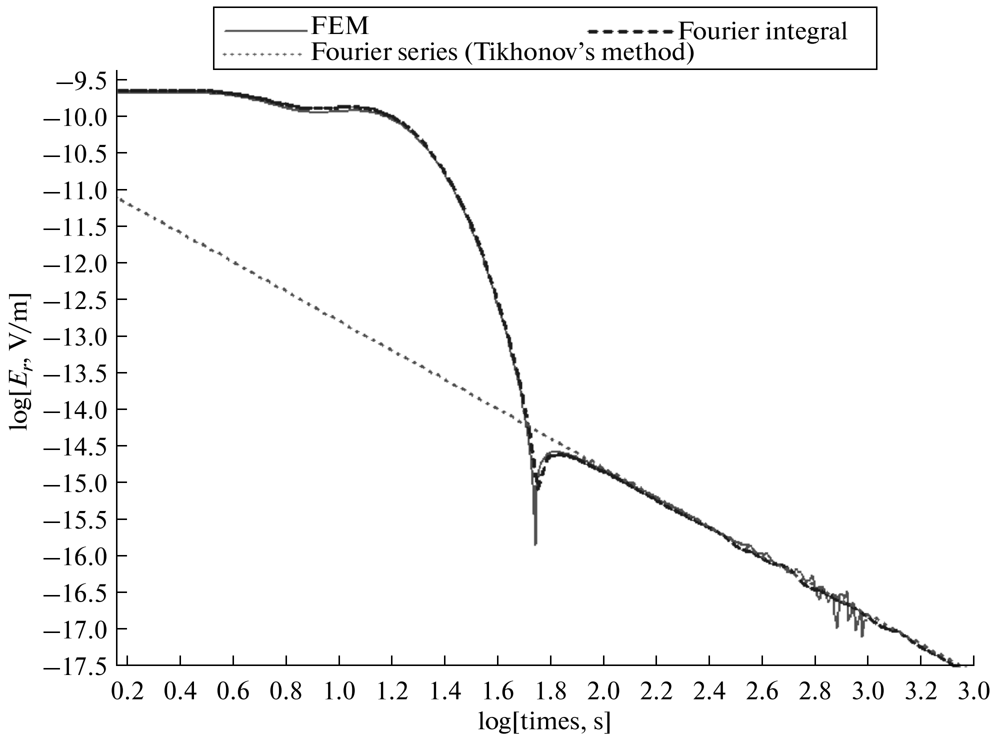

Fig. 9. The comparison of the calculations by the three methods.

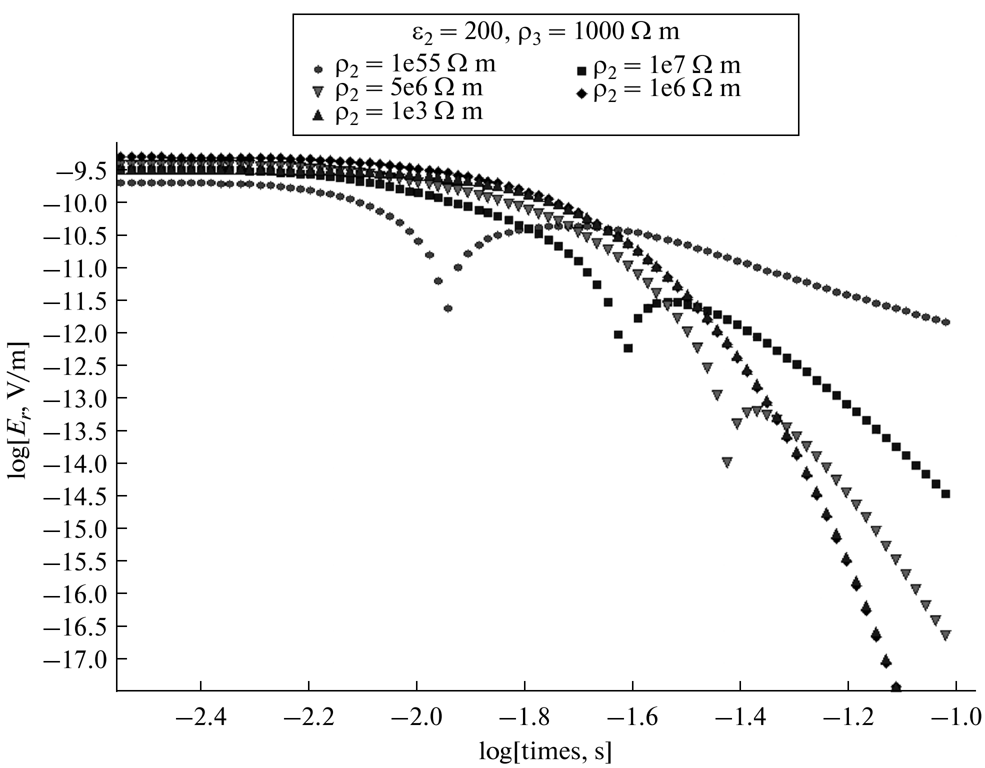

Fig. 10. The dependence on the resistivity of the high-ohmic layer.

PRACTICAL VALUE

Of course, the results shown in Figs. 6 and 7 are quite odd for electric prospecting. However, this immediately raises a question about the practical applicability of these effects. The real geological medium is never perfectly insulating. How does the finite conductivity of the screen affect the role of the displacement currents? Figure 10 illustrates this dependence. It can be seen that the decrease in the resistivity of the screen even to 1 000 000 Ω m leads to a situation when the curves for the cited resistivity and for 1000 Ω m (i.e., the screen blends with the underlying layer) are practically close at the late stage.

CONCLUSIONS

Thus, at the super-early stage of the transient process (in the shallow-depth soundings), the displacement currents can be vital for any field source. The mathematical apparatus of the common electrical prospecting which implies the solution in the frequency domain with its subsequent Fourier transformation should be profoundly modified in this case. The pattern of the wave processes is different for the TE- and TM-polarized fields. The key difference lies in the fact that the TM-source (a circular electric dipole or a vertical electric dipole) does not produce a wave in the air.

However, our studies also show that in the deep soundings in the media containing the insulating inclusions, the displacement currents can also play an important role if the excited field is TM-polarized. This result has fundamental value. At the same time, the finite conductivity of the real geological medium strongly reduces the topicality of this problem in practice. Besides, our result is obtained for a 1D model of a medium whose role is limited in practical applications.

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