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The magnetic relaxation effect on TEM responses of a uniform earth

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Abstract

Ungrounded horizontal loop transient responses of uniform conductive and magnetically viscous earth have been simulated using two different codes. One algorithm employs the relationship between viscous magnetization and the magnetic flux it induces in the receiver loop. In the other algorithm, the Helmholtz equation in a boundary-value problem is solved using the Fourier transform with frequency-dependent magnetic permeability. The two solutions are identical for noncoincident loops but differ when the transmitter and receiver loops are closely spaced (at 1 cm or less). In the latter case correct results are provided by the first code. The magnetic relaxation and eddy current responses appear to be independent at conductivities typical of the real subsurface. Therefore, TEM responses of magnetically viscous conductors can be computed using the superposition principle. Although transients change in an intricate way as a function of loop geometry and earth parameters, these changes exhibit certain patterns which may be useful at the stages of exploration and TEM data processing. In configurations where the receiver loop is laid outside the transmitter, the interaction of magnetic relaxation and eddy current decay causes sign reversal in transients. This reversal occurs at late times after an earlier sign reversal due uniquely to eddy current.

Keywords: TEM method; magnetic viscosity; superparamagnetism; conductivity; simulation

Introduction

Magnetic viscosity is a property of ferromagnetism. In rocks it is associated with superparamagnetism, or magnetic relaxation of ultrafine grains in ferrimagnetic minerals. Magnetic viscosity normally causes a much less effect on TEM data than eddy current. There are, however, natural and man-made objects in which the amount of superparamagnetic particles is as great as to make the magnetic relaxation response notable or even dominant over the conductivity-controlled eddy current response. This is the effect that cannot be ignored in data interpretation.

Magnetic viscosity is most often treated as geologic noise that interferes with TEM responses to be interpreted in terms of "normal" electrical conductivity (Buselli, 1982; Dabas and Skinner, 1993; Lee, 1984a,b; Pasion et al., 2002; Zakharkin et al., 1988; Zakharkin and Bubnov, 1995). On the other hand, there is evidence that magnetic viscosity effects in TEM measurements bear signature of genesis and structure of natural and man-made materials and near-surface processes (Barsukov and Fainberg, 1997, 2002; Kozhevnikov and Nikiforov, 1996, 1999; Kozhevnikov and Snopkov, 1990, 1995; Kozhevnikov et al., 1998, 2001, 2003). Therefore, it appears reasonable to learn how to (i) amplify or damp the magnetic viscosity response, (ii) image lateral and vertical magnetic viscosity profiles in shallow subsurface, and (iii) interpret the results in terms of rock physics and, possibly, magnetic mineralogy.

For this purpose, special tools are required for mathematical modeling of transient responses of magnetically viscous ground, in addition to laboratory and field experiments. The primary objective is to design forward modeling codes to be complemented in the long run with inversion algorithms.

An important contribution to the modeling experience belongs to T.J. Lee who derived analytical equations for transient responses of a conductive superparamagnetic ground (Lee, 1984b) and a thin superparamagnetic layer on top of a conductive nonmagnetic ground (Lee, 1984a). As Lee reported, the magnetic viscosity effect was stronger in coincident than in separate loops, especially, in the case where the superparamagnetic material was confined to a thin top layer of the ground (Lee, 1984a). Lee also showed that the coincident-loop transient response depended on the loop area

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as well as on the wire radius, and the wire-radius dependence was more evident in loops on a superparamagnetic ground (Lee, 1984a,b). Lee's equations (1984 a, b) derived for circular loops are critical for understanding the physics of superparamagnetism and are useful to predict the order of the expected effects but they hardly can make the basis for appropriate modeling.

Interest in the magnetic viscosity effect on TEM data has recently been rekindled in applications of UXO (unexploded ordnance) detection (Pasion et al., 2002). The models for UXO detection simulated in-loop transient responses of magnetically viscous materials obtained with a small circular receiver at the center of a relatively small circular transmitter. With small loop systems, the eddy current decay is usually so rapid that it has died out before the first time gate. However, with large square-loop systems in conductive terrain, which is a common case of TEM surveys, the conductivity effect can produce a pronounced response.

As far as we know, there has not been much literature on mathematical modeling of TEM responses of magnetically viscous ground. We failed to find publications that would discuss different models and compare their performance at different resistivity patterns and loop geometries. This modeling, however, will be an indispensable support to TEM soundings of a superparamagnetic ground which can make magnetic viscosity an inversion-derived parameter. We are trying to somehow bridge the gap by this study using the available literature on magnetic viscosity of rocks and our own results, which were partly reported elsewhere in brief communications (Antonov and Kozhevnikov, 2003; Kozhevnikov and Antonov, 2004).

Magnetic relaxation and its relation with induction transients

Assume that a transmitter of DC current I has been on indefinitely and the transmitter and receiver loops lie on nonmagnetic ground (Fig. 1). In this case, the magnetic flux Φ_0 induced in the receiver loop is $\Phi_0 = IM_0$, where M_0 is the coefficient of inductance between two loops on nonmagnetic half-space.

If there is a magnetic object in the loop vicinity, the primary magnetic field H_1 charges its any elementary volume with the magnetization J. The magnetized object induces the secondary magnetic field H_2 which adds $\Delta\Phi$ to the initial magnetism Φ_0. Correspondingly, M_0 changes for the value ΔM called introduced inductance. It either amplifies or reduces the initial inductance depending on loop geometry and magnetic susceptibility of the ground. Measuring ΔM can give information on the presence of a magnetic object and on its properties. The inductance that bears the effect of one or several magnetized objects (including magnetic half-space) is called effective inductance (M_e). It is convenient to write M_e as

$$M_e = \mu_e M_0, \quad (1)$$

where μ_e is the effective relative magnetic permeability which allows for the response of magnetic objects in the loop vicinity and is $\mu_e = 1$ in their absence. There is always such μ_e that (1) fulfills exactly in the case of a horizontally uniform magnetic earth; otherwise, (1) is approximate.

As the current is turned off instantly at time $t = 0$, the primary magnetic field disappears immediately. Assume that the conductivity of the object and its host is so small that eddy current and the secondary magnetic field it induces decay rapidly on a time scale of the experiment and cause no effect on magnetization, but viscous magnetization decays slowly. Magnetic relaxation excites synchronous secondary magnetic field H_2 which induces the voltage $e(t) = -\dfrac{d\Phi}{dt}$ in the receiver loop.

In the case of single-loop or coincident-loop excitation and measurement, the magnetic flux equation will include the loop inductance L instead of the mutual inductance M.

Rock magnetism is often expressed via the magnetic susceptibility κ instead of the permeability μ. In the SI system, relative μ and κ are related as

$$\mu = 1 + \kappa. \quad (2)$$

Correspondingly, their effective counterparts are related as

$$\mu_e = 1 + \kappa_e. \quad (3)$$

Therefore,

$$M_e = M_0(1 + \kappa_e). \quad (4)$$

The superparamagnetic decay on removal of the applied magnetic field is slow, and μ_e, κ_e, and M_e are thus time-dependent. Then, the magnetic flux can be written as Duhamel's integral:

$$\Phi(t) = I(t)\, M_e(0) + \int_{-\infty}^{t} I(\tau) \frac{dM_e(t-\tau)}{dt} d\tau.$$

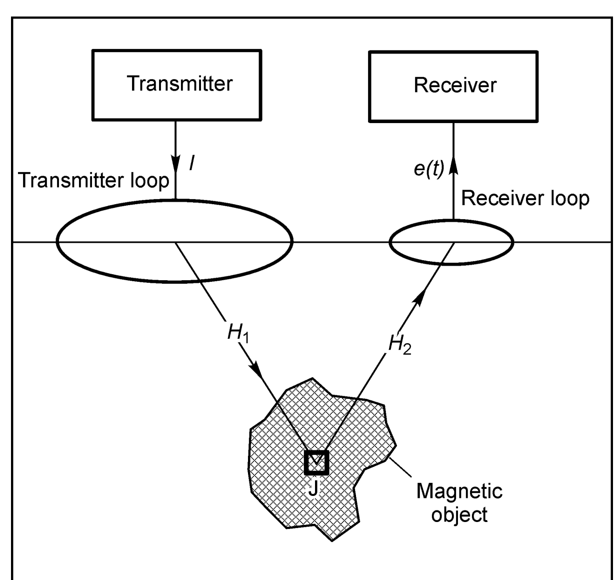

Fig. 1. Layout of TEM measurement system and magnetic object.

If the transmitter current I_0 is turned off at $t = 0$, the time-dependent current I in the loop is $I(t) = I_0[1 - 1(t)]$, where $1(t)$ is the unit Heaviside function. Then,

$$\Phi(t) = I(t)\,M(0) + I_0 \int_{-\infty}^{t} \frac{dM_e(t-\tau)}{dt}\,d\tau - I_0 \int_{-\infty}^{t} 1(t)\,\frac{dM_e(t-\tau)}{dt}\,d\tau. \quad (5)$$

The first integral in the right-hand side of (5)

$$\int_{-\infty}^{t} \frac{dM_e(t-\tau)}{dt}\,d\tau = M_e(t) - M_e(-\infty)$$

obviously gives static effective inductance (M_{es}), i.e., the inductance that sets up in an infinitely long time. The second integral in the right-hand side of (5) is

$$\int_{-\infty}^{t} 1(t)\,\frac{dM_e(t-\tau)}{dt}\,d\tau = \int_{0}^{t} \frac{dM_e(t-\tau)}{dt}\,d\tau = M_e(t) - M_e(0).$$

Inasmuch as viscous magnetization is zero at $t = 0$, $M_e(0) = M_0$, i.e., effective inductance equals initial inductance. Thus, (5) becomes

$$\Phi(t) = I_0 M_s - 1(t) I_0 M_0 - I_0 M_e(t),$$

while the voltage at the loop outputs is

$$e(t) = -\frac{d\Phi}{dt} = \delta(t) I_0 M_0 + I_0 \frac{dM_e(t)}{dt},$$

where $\delta(t)$ is the Dirac delta function. $M_e(t)$ can be expressed via the effective time-dependent magnetic susceptibility $\kappa_e(t)$ and the initial inductance M_0 as

$$M_e(t) = M_0[1 + \kappa_e(t)].$$

Therefore,

$$e(t) = -\frac{d\Phi}{dt} = \delta(t) I_0 M_0 + I_0 M_0 \frac{d\kappa_e}{dt}.$$

There the first term is the voltage induced in the receiver loop on the transmitter turn-off which bears no information on magnetic viscosity. The current-normalized voltage induced in the receiver by magnetic relaxation is

$$\frac{e(t)}{I_0} + M_0 \frac{d\kappa_e}{dt}. \quad (6)$$

To use (6) in practice, one has to calculate M_0 and specify the model for $\kappa_e(t)$.

Magnetic susceptibility of an ensemble of single-domain particles

An external magnetic field applied to a "normal" material magnetizes it immediately, i.e., the applied field H and the magnetization J are in phase and are related as $J = \kappa H$, where κ is time independent.

Magnetization of superparamagnetic materials is time-dependent. For a magnetic field applied at $t = 0$, $J(t) = \kappa(t)H$, where $\kappa(t)$ is time-dependent magnetic susceptibility. $J(t)$ is often written as

$$J(t) = \kappa_0 H\,[1 - P(t)], \quad (7)$$

where κ_0 is the static susceptibility and $P(t)$ is the after-effect function (Trukhin, 1973).

Magnetization of a single-domain grain has the relaxation time $\tau = \tau_0 \exp(KV/kT)$, where K is the anisotropy energy, V is the particle volume, T is the absolute temperature, k is Boltzmann's constant, and, $\tau_0 = 10^{-9}$ s (Neel, 1949). For a particle or an ensemble of particles with the same time constant, the after-effect function is $P(t) = \exp(-t/\tau)$ (Trukhin, 1973).

Relaxation times associated with superparamagnetic behavior of minerals in nature are in a range defined by the weight function $f(\tau)$ also called the distribution function. Then, the after-effect function is

$$P(t) = \int_{0}^{\infty} f(\tau) \exp(-t/\tau)\,d\tau. \quad (8)$$

The distribution of time constants in an ensemble of single-domain particles with uniformly distributed energy barriers between stable magnetization states is described by the Fröhlich function (Fannin and Charles, 1995). The relaxation times τ in this function are in the range from τ_1 to τ_2: $\tau_1 \le \tau \le \tau_2$. Inside the range,

$$f(\tau) = \frac{1}{\tau \ln(\tau_2/\tau_1)}, \quad (9)$$

and outside it $f(\tau) = 0$.

If the argument of (9) is $\ln \tau$, the Fröhlich function becomes $G(\ln \tau) = \frac{1}{\ln(\tau_2/\tau_1)}$, whence the relaxation times are uniformly distributed between τ_1 and τ_2.

Substituting (9) into (8) gives

$$P(t) = \frac{1}{\ln(\tau_2/\tau_1)} \int_{\tau_1}^{\tau_2} \frac{\exp(-t/\tau)}{\tau}\,d\tau. \quad (10)$$

The exact values of τ_1 and τ_2 are commonly unknown but this is of no significance in actual measurements. The range of time constants normally covers many orders of magnitude while magnetic relaxation is measured at $\tau_1 \ll t \ll \tau_2$. Then, the after-effect function is (Fannin and Charles, 1995)

$$P(t) = \frac{1}{\ln(\tau_2/\tau_1)} (-\gamma - \ln t - \ln \tau_2),$$

where $\gamma \approx 0.577$ is the Euler constant. Substituting (10) into (7) gives that the magnetization of a ground excited by a step external field increases proportionally to the logarithm of time:

$$J(t) = \frac{\kappa_0 H}{\ln(\tau_2/\tau_1)} [1 + A + \ln t],$$

where $A = \gamma + \ln \tau_2$. Dividing both sides of the equation by H gives the time-dependent susceptibility

$$\kappa(t) = \frac{\kappa_0}{\ln(\tau_2/\tau_1)}(1 - A + \ln t). \quad (11)$$

Note. The time-dependent susceptibility is meant in this study as a response to magnetic field turn-on and can be thus denoted $\kappa_{on}(t)$. This is an increasing time function. In some publications, it may correspond to a turn-off response and denoted $\kappa_{off}(t)$. Obviously, $\kappa_{off}(t) = \kappa_0 - \kappa_{on}(t)$.

Susceptibility of superparamagnetic materials was shown to be complex and frequency-dependent (Worm, 1999). In the frequency domain, the susceptibility of an ensemble of single-domain grains with their time constants satisfying (9) is given by (Fannin and Charles, 1995; Lee, 1984a,b; Trukhin, 1973)

$$\kappa^*(\omega) = \kappa_0\left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \cdot \ln\frac{(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)}\right], \quad (12)$$

where $j = \sqrt{-1}$, ω is the angular frequency.

The susceptibility of the form (12) approaches the static κ_0 at low frequencies and tends to zero at high frequencies. At frequencies $1/\tau_2 < \omega < 1/\tau_1$, the real component $\kappa^*(\omega)$ decreases proportionally to the logarithm of frequency and the imaginary component is frequency-independent (Fannin and Charles, 1995).

Uniform earth: effective permeability and transient response

Below we derive the equation for the transient response of a uniform conductive earth that also exhibits magnetic viscosity.

The inductance between two loops laid on a uniform ground with the relative permeability μ, as well as the inductance of each loop, increase by a factor of $2\mu/(\mu + 1)$, i.e., the effective (μ_e) and true (μ) relative permeabilities of a uniform half-space are related as (Spies and Frischknecht, 1991)

$$\mu_e = \frac{2\mu}{\mu + 1}. \quad (13)$$

Taking into account (2), (3) and (13), it is easy to show that the effective susceptibility κ_e of a uniform half-space is related to its true susceptibility κ as $\kappa_e = \kappa/(\kappa + 2)$.

For most geological conditions, $\kappa \ll 1$, and $\kappa_e = \kappa/2$.

The effective permeability of a ground with time-dependent susceptibility is likewise time-dependent, i.e., $\kappa_e(t) = \kappa(t)/2$, and, according to (6),

$$\frac{e(t)}{I_0} = \frac{1}{2} M_0 \frac{d\kappa(t)}{dt}.$$

With regard to (11), it becomes

$$\frac{e(t)}{I_0} = \frac{M_0}{2}\frac{\kappa_0}{\ln(\tau_2/\tau_1)\, t}\frac{1}{t}. \quad (14)$$

In order to use (14) for estimating the transient response of a uniform magnetically viscous ground, one has to calculate M_0 (L_0 for single-loop or coincident-loop configurations) and to set the static susceptibility κ_0 and the τ_2/τ_1 ratio.

Inversion becomes possible when measured transients are available. In (14) $e(t)$ is a measured value and M_0, I_0, and t are the derived values. Solving (13) with respect to κ_0 gives

$$\kappa_0 = \frac{2}{M_0}\ln\left(\frac{\tau_2}{\tau_1}\right)\frac{e(t)}{I_0}\, t. \quad (15)$$

Thus, measuring $e(t)$ can give κ_0. The susceptibility κ_0 equals the true static susceptibility in a uniform half-space and is an effective parameter in a nonuniform half-space, where it can be reasonably called apparent static susceptibility.

Note. The susceptibility κ_0 of a uniform half-space and the apparent static susceptibility of a nonuniform half-space are controlled by the logarithmic τ_2/τ_1 ratio in (14) and (15). Therefore, when reporting the results, one has to specify which τ_2 and τ_1 have been used in forward modeling or in inversion.

Algorithm based on the Fourier boundary-value solution

We consider a layered earth with the conductivities σ_1, ..., σ_i, ..., σ_N, permeabilities μ_1, ..., μ_i, ..., μ_N, and the layer interfaces at z_1, ..., z_i, ..., z_N, in the Cartesian coordinates xyz, where z is directed downward. The field of an arbitrary point source in a layered subsurface can be found from the spatial Fourier images of its vertical components (Tabarovsky, 1975).

A model loop system consists of horizontal electric dipoles placed along the contour of the transmitter loop. The voltage $e(t)$ induced in the receiver loop $L_r = L_r(\mathbf{r})$ on the removal of the transmitter field ($L_t = L_t(\mathbf{r}_0)$) is found by double integration:

$$\varepsilon = I \oint_{L_t}\oint_{L_r} E\,(Id\mathbf{l}_t, |\mathbf{r}_0 - \mathbf{r}|)\, dl_r\, dl_t,$$

where I is the current, $E(Id\mathbf{l}_t, |\mathbf{r}_0 - \mathbf{r}|)$ is the field of the electric dipole with the moment $Id\mathbf{l}_t$ located at the point $\mathbf{r}_0 = (x_0, y_0, z_0)$ of the transmitter loop L_t and measured at $\mathbf{r} = (x, y, z)$ of the receiver loop L_r. The transients generated by an inductive loop system can be calculated with regard to only the magnetic mode of the electrical point sources. The corresponding frequency-domain components of the electric field are given by

$$^{Ix}E_x = \frac{i\omega\mu_0 I}{4\pi}\frac{\partial^2}{\partial y^2}\int_0^\infty f(u, \omega, z, z_0)\, J_0(u|r - r_0|)\, u\, du, \quad (16)$$

$$^{Ix}E_y = {}^{Iy}E_x = \frac{i\omega\mu_0 I}{4\pi}\frac{\partial^2}{\partial x\, \partial y}\int_0^\infty f(u, \omega, z, z_0)\, J_0(u|r - r_0|)\, u\, du, \quad (17)$$

$$^{Iy}E_y = \frac{i\omega\mu_0 I}{4\pi}\frac{\partial^2}{\partial x^2}\int_0^\infty f(u, \omega, z, z_0)\, J_0(u|r - r_0|)\, u\, du. \quad (18)$$

There $u = \sqrt{k_x^2 + k_y^2}$, k_x, k_y, where k_x and k_y are the Fourier images of the spatial frequencies along the horizontal coordinates and J_0 is the zero-order Bessel function of the first kind.

The components of the randomly oriented source are expressed using a set of trigonometric functions. The integrand function $f(u, \omega, z, z_0)$ governs the field dependence on the earth parameters, including σ, μ, the interfaces (z_1, z_2, ..., z_{N-1}), the source and receiver depths z_0, z, and the frequency ω. The loop geometry is included in the Bessel function argument.

The function $f(u, \omega, z, z_0)$ is found using the recurrent formulas (Tabarovsky, 1979):

$$\alpha_N = 0,$$

$$\beta_{j-1} = \frac{\mu_j}{\mu_{j-1}} \frac{p_{j-1}}{p_j} \frac{\alpha_j + 1}{\alpha_j - 1},$$

$$R_{j-1} = \frac{1 + \beta_{j-1}}{1 - \beta_{j-1}},$$

$$\alpha_{j-1} = -e^{-2p_{j-1}(z_{j-1} - z_{j-2})} R_{j-1},$$

$j = N, ..., 2$, $p_j = \sqrt{u^2 + k_j^2}$, $k_j^2 = -i\omega\mu_j \sigma_j$, σ_j is the conductivity and $\mu = \mu(\omega)$ is the complex permeability. After p_1 and R_1 have been found, the function $f(u, \omega, z, z_0)$ is obtained as

$$f(u, \omega, z, z_0) = -\frac{e^{-p_1(2z_1 + z_0 - z)}}{2p_1} R_1 + \frac{e^{-p_1|z - z_0|}}{2p_1}.$$

Integrals (16)–(18) are calculated using special spline interpolation quadratures, and the quadrature coefficients for point sources are found once and saved in a special file. For an arbitrarily configured system, it is enough to integrate the quadrature coefficients for a point source. This approach speeds up computing the TEM responses for arbitrary loop geometries.

Software summary

The Unv_QQ program designed by E.Yu. Antonov in FORTRAN implements the above algorithm. It is applicable to compute the transient responses of magnetically viscous layered conductors for any rectangular-loop system. In addition to loop geometry and layer parameters, the user can specify the loop height above the earth surface. The magnetic viscosity effects are included using the frequency-dependent complex permeability $\mu^*(\omega) = \mu_0[1 + \kappa^*(\omega)]$, where $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the vacuum permeability, and $\kappa^*(\omega)$ is the susceptibility defined by (12).

The MVIS program designed by N.O. Kozhevnikov in the MATHCAD environment is used to compute transients with magnetic relaxation using equation (6) for the following loop configuration and earth models:

— loops of any geometry, uniform earth;

— circular coincident loops, two-layer earth;

— circular or square noncoincident in-loop system, layered earth with any number of layers.

Equation (6) was derived assuming a low-conductive subsurface in which eddy current decays very rapidly and, hence, causes no effect on the magnetic relaxation response. This may seem to be a limitation for MVIS making it restricted to high-resistivity, actually zero-conductivity, terrains. However, we show below that the eddy current and magnetic relaxation responses are independent in the conductivity range of real earth.

Time-dependent inductance and numerical method: comparing two approaches

At the first stage of the reported numerical experiments we compared the transients computed by the Unv_QQ and MVIS codes for the same loop systems and earth models. The results turned out to be identical for in-loop but different for coincident-loop transients.

To understand the cause of this difference, we computed in-loop transients for a system with a receiver loop of a side length varying from 90 to 100 m placed inside a 100-m square transmitter loop, i.e., a noncoincident-loop configuration graded into a coincident one.

We used a uniform earth model with a resistivity $\rho = 10^6$ Ohm·m and $\kappa_0 = 10^{-3}$ SI units. At this resistivity the effect of fast-decaying eddy current is vanishing relative to the magnetic relaxation response. It was assumed, both in calculation by (14) and in numerical experiments, that $\tau_1 = 1 \cdot 10^{-6}$ s, $\tau_2 = 1 \cdot 10^6$ s, and delay times between 10 μs and 100 ms, i.e., $\tau_1 \ll t \ll \tau_2$.

The inductance M_0 between the square loops was found using numerical integration of (3.12) from (Nemtsov, 1989). The loop inductance L_0 for coincident-loop transients was obtained with the equation from (Zimin and Kochanov, 1985, p. 163).

See the e_1/e_2 curves computed using MVIS and Unv_QQ in Fig. 2, where e_1 and e_2, respectively, are current-normalized. The two solutions diverged notably when the receiver loop became in close proximity of the transmitter (at about 1 cm). As they further approached one another, the e_1/e_2 ratio increased to become 1.9 in the limit, when the two loops coincided. The e_1/e_2 ratio turned out to be independent of time delay within the range from 10 μs to 100 ms.

Inasmuch as the two solutions were identical at loop spacing over 1 cm, we reasonably supposed that both approaches drove at the correct result, taking into account that they based on different methods.

What happens when the loops become closely spaced? See Fig. 2, *b* for the M_0 behavior of two square loops centered on the same point. The size of the outer loop remains invariable (100×100 m) while the inner loop grows from 1×1 m to 100×100 m, and M_0 increases proportionally to the loop area. As the loops approach one another, M_0 grows ever more rapidly and reaches its maximum when they coincide. Then, M_0 becomes L_0 of the 100-m loop which, unlike M_0, depends on both the loop size and the wire thickness. At a wire radius of 2 mm, $L_0 = 8.24 \cdot 10^{-4}$ H. Then, M_0 reaches half the L_0 when the receiver loop is 99.5×99.5 m. To put it different, inductance between the two loops becomes half the maximum value when they are spaced at only 25 cm.

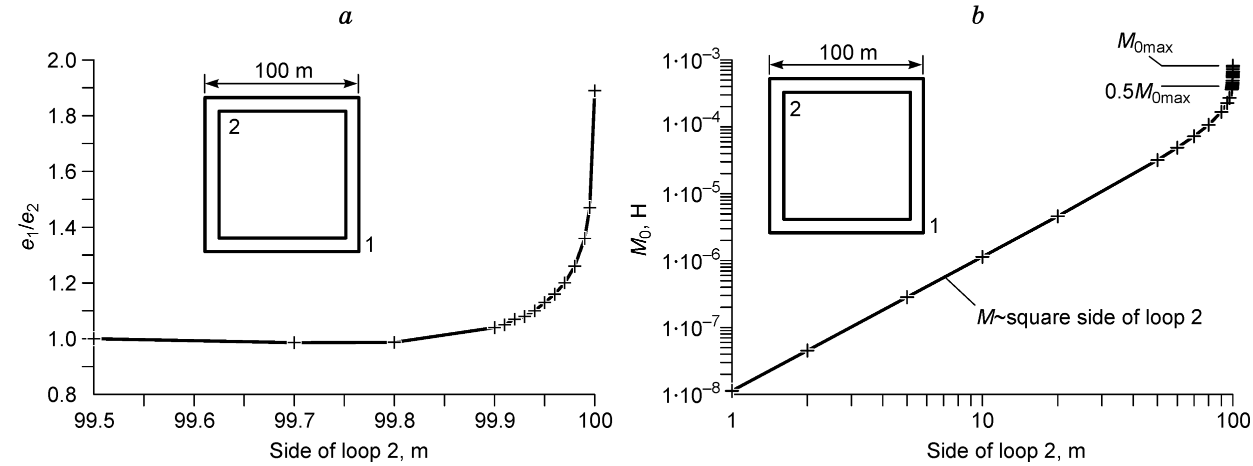

Fig. 2. e_1/e_2 ratio (*a*) and inductance M_0 between outer (1) and inner (2) loops (*b*) as a function of inner loop size.

According to (14), the magnetic relaxation response is proportional to the initial inductance M_0 of the transmitter and receiver loops. M_0 grows rapidly when the loops approach to a distance of a few centimeters and less (Fig. 2, *b*) and is the greatest in coincident loops. Therefore, coincident-loop systems are the most sensitive to magnetic viscosity.

The voltage induced in the receiver loop by eddy current is proportional to inductance between the receiver and the ground eddy current which can never coincide with the transmitter loop in any conditions. Eddy current diffusion can be illustrated by an equivalent current smoke-ring that grows in size and in depth (Nabighian, 1979). There is inductance between any two elementary ring filaments controlled by their size and position and by the ground magnetic properties. This interaction and, hence, the ground magnetic properties, are taken into account in the Unv_QQ solution. However, the eddy current maximum is far from the loop wire already at the earliest times. Therefore, the Unv_QQ solution does not include magnetization of ground in the immediate loop vicinity.

Results and discussion

Having explored the potentialities and the limitations of the two approaches, we discuss some results that may be useful in exploration and in processing the TEM responses of a magnetically viscous ground.

Figure 3 shows noncoincident-loop transient responses of a uniform earth measured with a 50-m square receiver inside a 100-m transmitter. First we computed the eddy current induced voltage $e_1(t)/I$ assuming $\rho = 10$, 10^2, and 10^3 Ohm·m, $\kappa_0 = 0$ (no magnetic viscosity).

Then we found the magnetic relaxation response $e_2(t)/I$ assuming $\kappa_0 = 0.001$ and the resistivity $\rho = 10^6$ Ohm·m at which eddy current decays so fast that the transient is controlled uniquely by magnetic viscosity.

Finally, we obtained the total response of eddy current plus magnetic relaxation. See the $e_\Sigma(t)/I = e_1(t)/I + e_2(t)/I$ curves in Fig. 3, together with the $e(t)/I$ curves computed using Unv_QQ with regard to eddy current–magnetic relaxation interaction, at $\kappa_0 = 0.001$, $\rho = 10$, 10^2, and 10^3 Ohm·m. The $e_\Sigma(t)/I$ and $e(t)/I$ curves coincide, which proves the independence of the magnetic relaxation and eddy current responses (Kozhevnikov and Snopkov, 1990, 1995). Therefore, the superposition principle is applicable to computing and interpreting transients of magnetically viscous conductors.

The resistivity ρ in early-time transients controls the eddy current response (Fig. 3) while at late times the curves follow the asymptote corresponding to the magnetic relaxation response. The higher the resistivity the earlier the time when the curves turn to and reach the asymptote. The transient process in nonconductive earth is obviously controlled uniquely by its magnetic viscosity. The decay of magnetization is $1/t$, i.e., is much slower than in the eddy current response. The magnetic viscosity effect shows up in the behavior of apparent resistivity (ρ_τ) derived from the transients (Fig. 4) as a continuous ρ_τ fall-off to the $1/t$ asymptote. The higher the resistivity, the earlier the fall-off becomes notable.

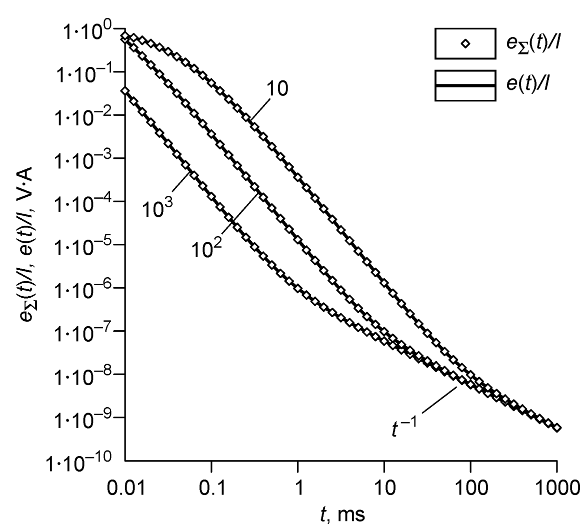

Fig. 3. 100×50 m in-loop transient responses of uniform earth: approximate and exact solutions. Figures on curves are resistivities in Ohm·m. For all models, $\kappa_0 = 0.001$ SI units.

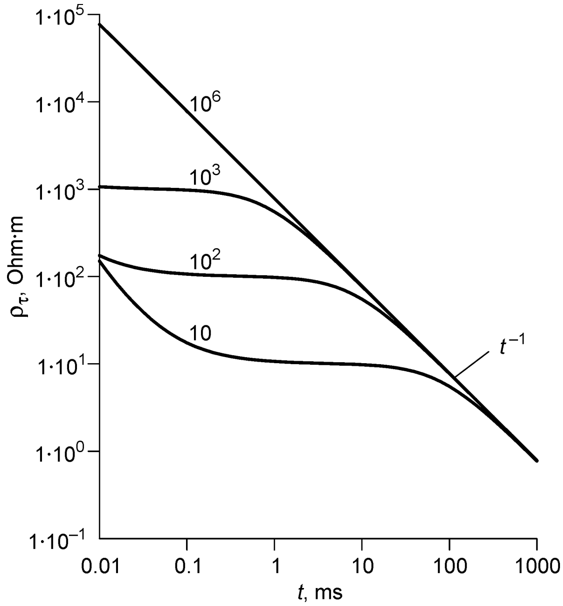

Fig. 4. Apparent resistivity curves for 100×50 m in-loop configuration, uniform earth. Figures at curves are resistivities in Ohm·m. For all models, $\kappa_0 = 0.001$ SI units.

It is essential to figure out how the magnetic viscosity effect depends on the loop geometry and size in TEM soundings of a ground that may be superparamagnetic. Figure 5 shows in-loop transient responses of a uniform earth with $\rho = 10^3$ Ohm·m, $\kappa_0 = 10^{-3}$ SI units, $\tau_1 = 10^{-6}$ s, $\tau_2 = 10^6$ s. The transmitter loop side varied from 10 to 10^3 m and the receiver was always 10×10 m.

As the loop size increased, the eddy current effect increased as well but the magnetic viscosity effect decreased (Fig. 5, *a*). Mind that the magnetic relaxation response is proportional to the transmitter-receiver inductance. At an invariable receiver size, this induction is the greatest when the loops coincide (see Fig. 5, *b* for the corresponding apparent resistivity curves). Therefore, the magnetic viscosity effect can be highlighted by using small coincident loops and damped, if unwanted, in noncoincident loop transients, with a small receiver inside a large transmitter.

If we assume that the inductance is positive ($M_0 > 0$) for a receiver inside a transmitter, it will be negative ($M_0 < 0$) if the receiver is placed outside the transmitter (Nemtsov, 1989). The magnetic viscosity-controlled transient has the same sign as a "normal" transient in the former case and there is a sign reversal in the latter case (Fig. 6, *a*). The system of Fig. 6, *a* consists of a 50-m square receiver loop placed outside a 100-m square transmitter loop, at a distance of 80 m between the loop centers; the loops lie on a uniform ground with $\rho = 10^3$ Ohm·m and $\kappa_0 = 10^{-3}$ SI units. See a sign reversal at about 1 ms caused by interaction of eddy current decay and magnetic relaxation (Fig. 7). When the transmitter is on, it produces a

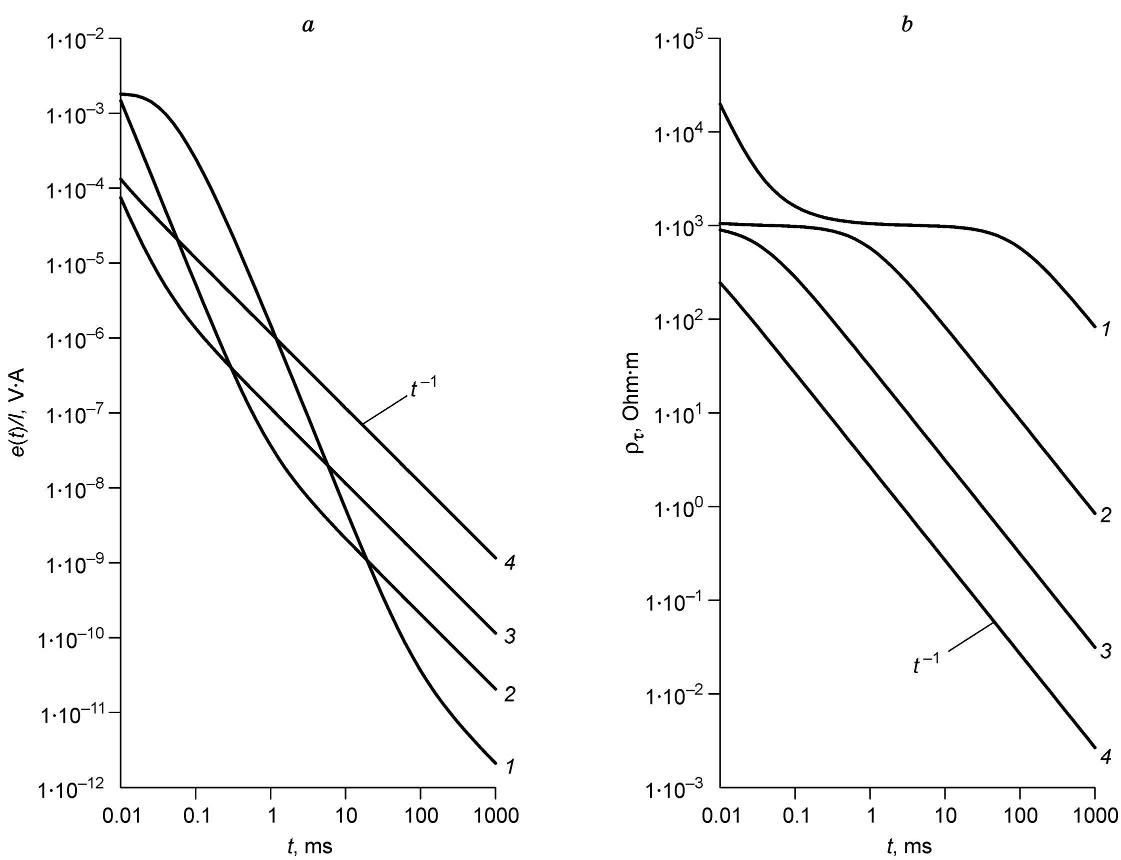

Fig. 5. In-loop transients (*a*) and apparent resistivity curves (*b*), uniform earth ($\rho = 10^3$ Ohm·m, $\kappa_0 = 0.001$ SI units). Receiver loop is 10×10 m in all cases; transmitter loop is 1000×1000 m (*1*), 100×100 m (*2*), 20×20 m (*3*), and 10×10 m (*4*).

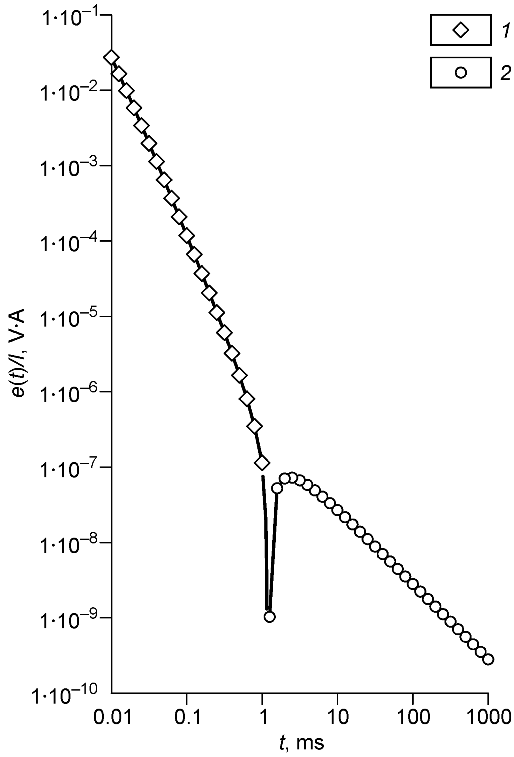

Fig. 6. Noncoincident-loop transient response of uniform earth ($\rho = 10^3$ Ohm·m, $\kappa_0 = 0.001$ SI units). The centers of a 100-m square transmitter loop and a 50-m receiver loop are spaced at 80 m. See sign reversal from positive (*1*) to negative (*2*).

primary magnetic field that causes the magnetization J in rocks. In an isotropic earth, at $\kappa_0 \ll 1$, the direction of this magnetization follows that of the primary transmitted field at any point of the subsurface.

The removal of the transmitter field launches two processes: magnetic relaxation and eddy current decay. Magnetic relaxation is simultaneous at all points of the subsurface, i.e., the magnetization ratio of any two points remains invariable. This ratio is defined by the transmitter field and earth's $\kappa_0(x, y, z)$ patterns. See that the secondary magnetic field always aligns with the primary field everywhere at the ground surface (Fig. 7).

An eddy current response illustrated by an equivalent current smoke-ring (Nabighian, 1979) is as follows. First it occurs under the transmitter loop and is approximately of the same size, and the magnetic field it induces inside (point 1) and outside (point 2) the transmitter aligns with the primary field. With the time on, the ring diffuses laterally and depthward, and once it grows to the size when the surface projection of the current line falls at point 2, the magnetic field induced by eddy current changes its polarity. (In our case, the sign reversal occurs at less than 10 μs and is beyond the transient of Fig. 6.)

The field eddy current induces around point 2 is greater than the superparamagnetically induced field of opposite direction. However, the field induced by the current ring at point 2 decreases as the eddy current diffuses down and laterally decaying by heat loss. Inasmuch as magnetic relaxation is much slower than the decay of eddy current, its field will exceed that of eddy current since some moment of time. That is the point of sign reversal of the total field which keeps decaying to finally reach its minimum; then it decays to zero remaining negative. The transient experiences sign reversal at the point when the total field is minimum and then remains negative decaying further as $1/t$ (Fig. 6, *a*).

Above we mentioned that in noncoincident loop configurations $M_0 > 0$ when the receiver is inside the transmitter and $M_0 < 0$ when it is laid outside. Thus, there always must be a receiver position corresponding to $M_0 = 0$ (Nemtsov, 1989), i.e., the magnetic viscosity effect becomes eliminated in a magnetically uniform earth. Inasmuch as TEM measurements

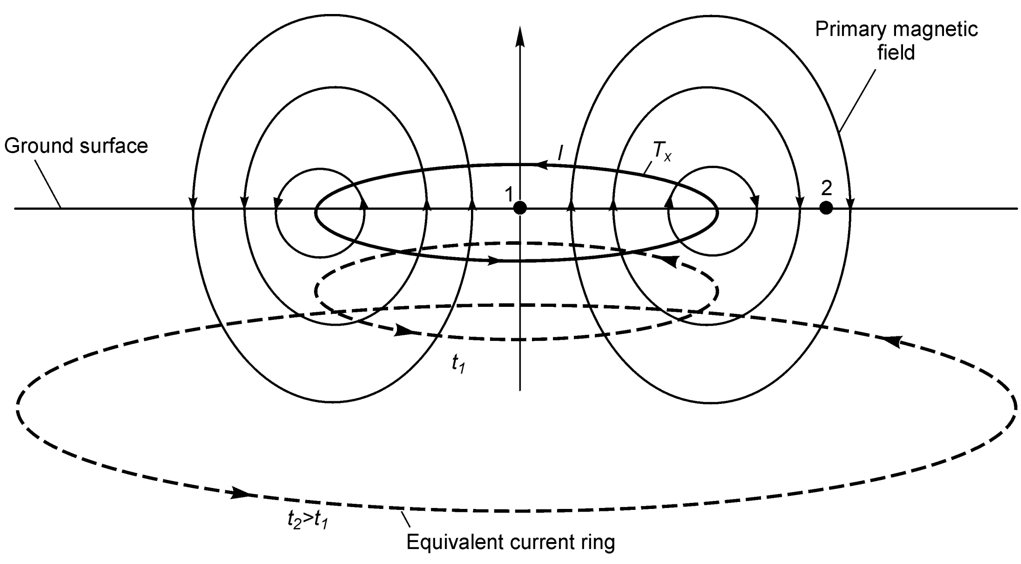

Fig. 7. Transmitter loop on ground surface, its primary magnetic field, and equivalent current ring. Magnetic relaxation and its secondary field are along primary field at any point of ground surface.

are most often taken at late times, the displacement of the transmitter loop causes no influence on the eddy current response.

Conclusions

Due regard for magnetic viscosity of the ground is a topical problem in TEM surveys. We modeled the magnetic relaxation effect on transient responses of uniform earth with two algorithms. One code employed the relationship between viscous magnetization and the magnetic flux it induces in the receiver loop. This is a simple solution, easy to illustrate physically, but it is not rigorous as it neglects interaction between eddy current and magnetic relaxation.

The other algorithm was based on the numerical solution to the Helmholtz equation taking into account the eddy current–magnetic viscosity interaction. Transient responses computed with the two codes for the same loop configurations and earth models were identical if the transmitter and receiver loops were spaced at more than a few centimeters but differed when the spacing reduced to 1 cm and less. Therefore, both methods provided correct results for noncoincident-loop configurations while the former algorithm was more workable in the case of coincident loops. The magnetic relaxation and eddy current responses were shown to be independent at conductivities common to the real subsurface. Therefore, TEM responses of magnetically viscous conductors can be computed using the superposition principle. Transient responses of a magnetically viscous conductive earth changed in an intricate way as a function of loop geometry and earth parameters but the changes exhibited certain patterns which may be useful at the stages of exploration and TEM data processing

In noncoincident loop configurations, with a receiver outside the transmitter, the interaction of magnetic relaxation and eddy current decay caused sign reversal in transients. The reversal occurred after an earlier sign reversal due uniquely to the eddy current decay.

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References

Antonov, E.Yu., Kozhevnikov, N.O., 2003. Simulating the magnetic viscosity effect on ungrounded loop transients, in: Geophysical Methods in Prospecting and Environmental Applications. Proc. Russian R&D Conf. [in Russian], Izd. TPU, Tomsk, pp. 51–55.

Barsukov, P.O., Fainberg, E.B., 1997. A "chimney" superparamagnetic effect over gold and nickel deposits. Dokl. RAN 353 (6), 811–814.

Barsukov, P.O., Fainberg, E.B., 2002. TEM soundings of environment with regard to IP and SPM effects. Izv. RAN, Ser. Fizika Zemli 11, 82–85.

Buselli, G., 1982. The effect of near-surface superparamagnetic material on electromagnetic transients. Geophysics 47 (9), 1315–1324.

Dabas, M., Skinner, J.R., 1993. Time-domain magnetization of soils (VRM), experimental relationship to quadrature susceptibility. Geophysics 58 (3), 326–333.

Fannin, P.C., Charles, S.W., 1995. On the influence of distribution functions on the after-effect function of ferrofluids. J. Phys. D. Appl. Phys. 28, 239–242.

Kozhevnikov, N.O., Antonov, E.Yu., 2004. Modeling TEM response of a magnetically-viscous conductive ground. 66th EAGE Conference, Expanded Abstracts. Paris, France, P088.

Kozhevnikov, N.O., Kharinsky, A.V., Kozhevnikov, O.K., 2001. An accidental geophysical discovery of an Iron Age archaeological site on the western shore of Lake Baikal. J. Appl. Geophys. 47 (2), 107–122.

Kozhevnikov, N.O., Kozhevnikov, O.K., Kharinsky, A.V., 1998. How search for a geophysical solution drove at a discovery of an archaeological site. Geofizika 6, 48–60.

Kozhevnikov, N.O., Kozhevnikov, O.K., Kharinsky, A.V., Urbat, M., 2003. Chemical composition and magnetism of ancient metallurgical slags from the Chernorud site on the western shore of Lake Baikal. Archaeometallurgy in Europe. Proc. Int. Conf. Milan, Italy, 1, 525–534.

Kozhevnikov, N.O., Nikiforov, S.P., 1996. Magnetic viscosity of baked clays and the possibility of its use in the location of buried ceramic objects. Proc. SAGEEP'96, Keystone, Colorado, 499–505.

Kozhevnikov, N.O., Nikiforov, S.P., 1999. Magnetic viscosity of sintered clay and its applicability to archaeology. Rossiiskii Geofizicheskii Zhurnal 13–14, 42–46.

Kozhevnikov, N.O., Snopkov, S.V., 1990. Superparamagnetism in Resistivity Surveys [in Russian]. Deposited in VINITI 13.08.90, N 4584-V90, Irkutsk.

Kozhevnikov, N.O., Snopkov, S.V., 1995. Supermagnetism of traps and its relation to TEM anomalies (Yakutian Kimberlite Province). Geologiya i Geofizika (Russian Geology and Geophysics) 36 (5), 91–102 (89–100).

Lee, T.J., 1984a. The effect of a superparamagnetic layer on the transient electromagnetic response of a ground. Geophys. Prosp. 32, 480–496.

Lee, T.J., 1984b. The transient electromagnetic response of a magnetic or superparamagnetic ground. Geophysics 49 (7), 854–860.

Nabighian, M.N., 1979. Quasi-static transient response of a conducting half-space: An approximate representation. Geophysics 44, 1700–1705.

Néel, L., 1949. Théorie du trainage magnetique des ferromagnetiques en grains fins avec application aux terres cuites. Ann. Geophys. 5, 99–136.

Nemtsov, M.V., 1989. Calculating Parameters of Inductance Coils. A Handbook [in Russian], Energoatomizdat, Moscow.

Pasion, L.R., Billings, S.D., Oldenburg, D.W., 2002. Evaluating the effects of magnetic soils on TEM measurements for UXO detection. Expanded Abstracts. Society of Exploration Geophysicists, Tulsa, OK, 1428–1431.

Spies, B.R., Frischknecht, F.C., 1991. Electromagnetic sounding. Electromagnetic methods in Applied Geophysics (SEG) 2, 285–386.

Tabarovsky, L.A., 1975. The Method of Integral Equations as Applied to Geoelectrical Problems [in Russian]. Nauka, Novosibirsk.

Tabarovsky, L.A., 1979. Transversal electric and transversal magnetic fields in layered media, in: Antonov, Yu.N. (Ed.), Induction Logging Methods [in Russian]. Nauka, Novosibirsk, pp. 225–233.

Trukhin, V.I., 1973. Introduction into Rock Magnetism [in Russian]. Moscow University Press, Moscow.

Worm, H.-U., 1999. The superparamagnetism of Yucca Mountain Tuff. J. Geophys. Res. 104, 25,415–25,425.

Zakharkin, A.K., Bubnov, V.M., 1995. A Geophysical Prospecting Tool. Invention Certificate 2045083 of the Russian Federation [in Russian]. Published in BI, 27.09.95, N 27.

Zakharkin, A.K., Bubnov, V.M., Kryzhanovsky, V.A.,Tarlo, N.N., 1988. Magnetic viscosity of rocks as a new complicating effect on TEM soundings, in: Surkov, V.S. (Ed.), TEM Survey for Mineral Prospecting in Siberia [in Russian], Nauka, Novosibirsk, pp. 19–26.

Zimin, E.F., Kochanov, E.S., 1985. Measurements of Electrical and Magnetic Parameters in Conductive Earth [in Russian]. Energoatomizdat, Moscow.

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